Angle Modulation

- There is another method of modulating a sinusoidal carrier wave, namely, angle modulation in which either the phase or frequency of the carrier wave is varied according to the message signal.
In this modulation method, the amplitude of carrier is maintained constant.

Angle modulation
- Frequency Modulation (FM)
- Phase Modulation (PM)

• frequency of carrier is varied according to the message signal.

• Phase angle of the carrier is varied according to the message signal.

let carrier signal, $c(t) = A_c \cos(2\pi f_c t + \underbrace{\phi}_{\text{phase}})$

$$2\pi f_c t + \phi = \theta(t) = \text{total angle}.$$

Phase Modulation :-

$$\phi \propto m(t)$$

or, $\phi = k_p m(t)$; where k_p = phase modulation constant

unit of k_p = radian/volt (or)
= radian/amp.

$$\theta(t) = 2\pi f_c t + k_p m(t).$$

$$\therefore \underbrace{\boxed{s(t) = A_c \cos(2\pi f_c t + k_p m(t))}}_{\text{General expression of PM signal}}$$

Instantaneous frequency in PM case:-

$$\omega_i(t) = \frac{d\phi_i(t)}{dt} \qquad \theta(t) = 2\pi f_c t + k_p m(t) = \omega_c t + k_p m(t)$$

$$\omega_i(t) = \frac{d\theta_i(t)}{dt} = \omega_c + k_p \frac{dm(t)}{dt}$$

$$\Rightarrow 2\pi f_i = 2\pi f_c + k_p \frac{dm(t)}{dt}$$

$$\Rightarrow \boxed{f_i = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}}$$

frequency modulation :-

Instantaneous frequency in FM :-

$f_i = f_c + k_f m(t)$ → message signal.

f_c ↓ carrier frequency

k_f → frequency sensitivity

→ k_f (Hz/volt) or (rad/sec/volt)

Now, $\omega_i = \frac{d\theta_i(t)}{dt}$ (relation b/w angular frequency & angle).

$$\Rightarrow 2\pi f_i = \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = \cancel{2\pi f f_i dt}\; 2\pi \int f_i \, dt$$

$$\Rightarrow \theta_i(t) = 2\pi \int \{f_c + k_f m(t)\} \, dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t) \, dt$$

$$\therefore \boxed{s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\, dt\right)}$$

is General expression of FM signal.

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As, $f_i = f_c + k_f m(t)$

if $m(t) = 0$, $f_i = f_c$

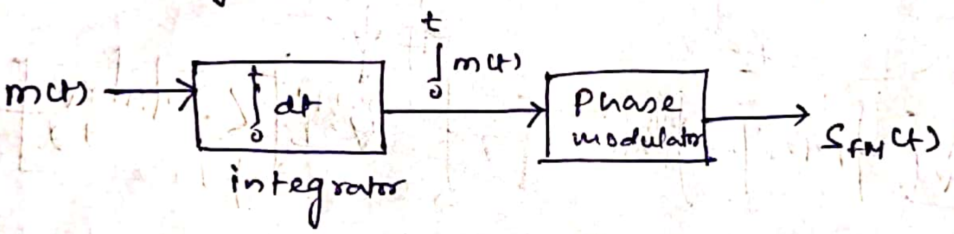
$m(t) = +ve$, ~~for~~ $f_i > f_c$

$m(t) = -ve$, $f_i < f_c$.

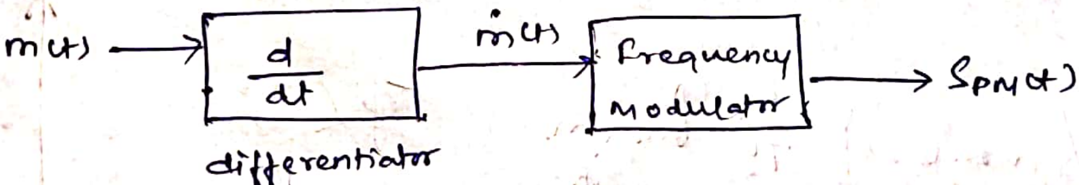
Here, In FM, message signal variations are converted to as carrier signal frequency variation, so it is called as "voltage to frequency conversion".

Relation b/w FM and PM :-

FM using PM:-



PM using FM:-



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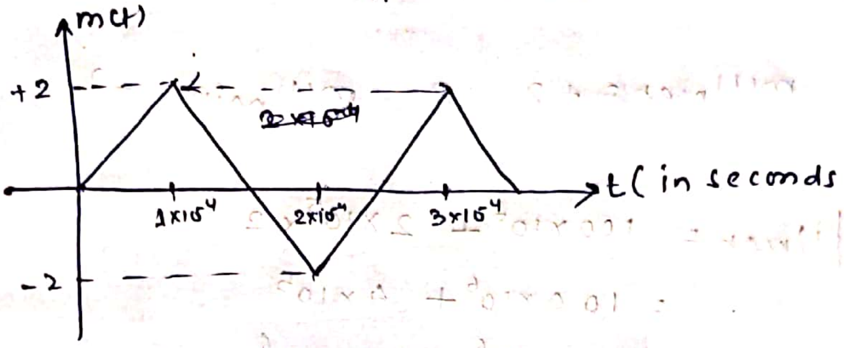
Example 1 :-

[this type of questions come in GATE Exam)

for the message signal shown below calculate the max^m and min^m value of instantaneous frequency if the message signal is (a) phase modulator (b) frequency modulator. ($k_f = 2\pi \times 10^5$ rad/sec/v and $k_p = 10\pi$ rad/V)
($f_c = 100$ MHz)



Sol^n :- for (a) phase modulation.

$$f_i = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

$$(f_i)_{max} = f_c + \frac{k_p}{2\pi}\underbrace{\left(\frac{dm(t)}{dt}\right)_{max}}_{\text{slope.}}$$

$$(f_i)_{min} = f_c + \frac{k_p}{2\pi}\left(\frac{dm(t)}{dt}\right)_{min}$$

$$m_1 = \text{slope} = \frac{+2-(-2)}{3\times10^{-4} - 2\times10^{-4}} = \frac{4}{10^{-4}} = 4\times10^4 \text{ (max)}$$

$$m_2 = \frac{-2-(+2)}{2\times10^{-4} - 1\times10^{-4}} = \frac{-4}{10^{-4}} = -4\times10^4 \text{ (min)}$$

$$\therefore (f_i)_{max} = 100\times10^6 + \frac{\cancel{10\pi}^{\,5}}{\cancel{2\pi}}\times 4\times10^4$$

$$= 10^8 + 20\times10^4$$

$$= 100\times10^6 + 0.2\times10^6$$

$$= 100.2 \text{ MHz}$$

Ily, $(f_i)_{min} = 100\times10^6 - 0.2\times10^6 = 99.8$ MHz.

(b) For (B) frequency modulator.

$f_i = f_c + k_f m(t)$

$\therefore (f_i)_{max} = f_c + k_f [m(t)]_{max}$

$(f_i)_{min} = f_c + k_f [m(t)]_{min}$

Here, $m(t)_{max} = +2$ & $m(t)_{min} = -2$

$\therefore (f_i)_{max} = 100\times10^6 + 2\times10^5 \times 2$

$= 100\times10^6 + 4\times10^5$

$= 100\times10^6 + 0.4\times10^6$

$= 100.4$ MHz

Hly, $(f_i)_{min} = 100\times10^{+6} - 0.4\times10^6$

$= 99.6$ MHz.

Single-tone Frequency modulation :-

let $m(t) = A_m \cos 2\pi f_m t$

$S_{FM}(t) = A_c \cos\{2\pi f_c t + 2\pi k_f \int m(t)dt\}$

$= A_c \cos\left\{2\pi f_c t + 2\cancel{\pi} k_f \times \frac{A_m \sin 2\pi f_m t}{2\cancel{\pi} f_m}\right\}$

$= A_c \cos\left\{2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right\}$

β = modulation index

$\beta = \frac{k_f A_m}{f_m}$

Now, $f_i = f_c + k_f m(t)$

Here, $m(t) = A_m \cos 2\pi f_m(t)$, $m(t)_{max} = A_m$

$\therefore (f_i)_{max} =$ $m(t)_{min} = -A_m$

$\therefore (f_i)_{max} =$ max^m frequency of FM signal $= f_c + k_f A_m$

$(f_i)_{min} =$ minim^m " " " " $= f_c - k_f A_m$.

So, max^m frequency deviation = ~~fc±k~~

$$\Delta f = \max\{k_f\, m(t)\}$$
$$= k_f A_m.$$

$$\therefore \boxed{\begin{aligned} f_{max} &= f_c + \Delta f \\ f_{min} &= f_c - \Delta f \end{aligned}}$$

$\therefore$ total frequency swing of FM signal $= f_{max} - f_{min}$
$= 2\Delta f$

Now, coming to β,

$$\boxed{\beta = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m}}$$

$\Delta f \rightarrow$ frequency deviation
$f_m \rightarrow$ message frequency.

$$\therefore \boxed{S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}}$$

$\rightarrow$ single-tone FM expression.

FM

$\beta \leq 1$ (NBFM) Narrow-band FM

$\beta > 1$ (WBFM) wide-band FM.

Narrow-band FM :-

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$$= A_c \{\cos 2\pi f_c t \cdot \cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - \sin 2\pi f_c t \sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta})\}$$

Here, β is small for NBFM,

$\therefore$ θ is also small. (as sine value can only be b/w 0 & 1)

And, we know that

for small θ,

$\cos\theta = 1$

$\sin\theta = \theta$

$\therefore$

$$S_{FM}(t) = A_c\{\cos 2\pi f_c t - \sin 2\pi f_c t \times \beta \sin 2\pi f_m t\}$$

$$= A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin 2\pi f_m t$$

$$S_{FM}(t) = A_c \cos 2\pi f_c t - \frac{A_c \beta}{2}\{\cos 2\pi(f_c - f_m)t\} + \frac{A_c\beta}{2}\{\cos 2\pi(f_c + f_m)t\}$$

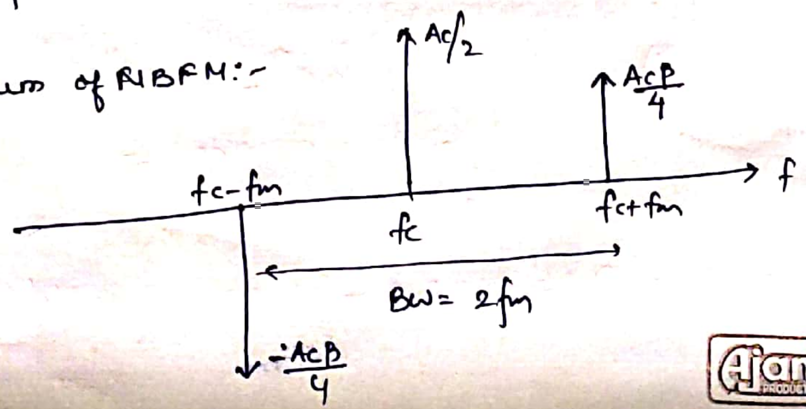
both has value ≤ 1.

Comparing this with AM expression:-

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi(f_c - f_m)t + \frac{A_c \mu}{2} \cos 2\pi(f_c + f_m)t$$

Note:- General expression of AM and NBFM are same except 180° phase shift at LSB frequency component.

Spectrum of NBFM:-



Power of NBFM :-

As $P_t = P_c + P_{LSB} + P_{USB}$

$$P_c = \frac{A_c^2}{2R}, \quad P_{LSB} = \frac{A_c^2\beta^2}{8R}, \quad P_{USB} = \frac{A_c^2\beta^2}{8R}$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{8R} + \frac{A_c^2\beta^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2\beta^2}{4R}$$

$$\therefore P_t = \frac{A_c^2}{2R}\left[1 + \frac{\beta^2}{2}\right]$$

$$\therefore \boxed{P_t = P_c\left(1 + \frac{\beta^2}{2}\right)}$$

similar to AM.

Note :- NBFM has much similarity with AM, hence practical ~~signifan~~ significance of NBFM is negligible

→ Bandwidth and power requirements are same as AM.

Wideband FM :-

Before continuing with WBFM, first we have to learn about Bessel function in communication system.

Bessel function:-

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)} \, d\theta \qquad \theta = \text{dummy Variable.}$$

* Properties of $J_n(x)$:-

1. $J_n(x) \downarrow$ as $n \uparrow$.

So, $\boxed{J_0(x) > J_1(x) > J_2(x) \cdots\cdots}$

2. $J_n(+x) = (-1)^n J_n(x)$

So, $\boxed{\begin{cases} J_{-n}(x) = -J_n(x) & ; n = \text{odd} \\ J_{-n}(x) = J_n(x) & ; n = \text{even} \end{cases}}$

3. $$\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$$

4. $J_n(x)$ is a real quantity.

Now,

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

Now, $e^{j\theta} = \cos\theta + j\sin\theta$

$\therefore$ $\cos\theta = Re\{e^{j\theta}\}$ and $\sin\theta = Im\{e^{j\theta}\}$

So, $S_{FM}(t) = A_c\, Re\{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\}$

$$= A_c\, Re\{e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\} \quad — (1)$$

Here, $e^{j\beta \sin 2\pi f_m t}$ is a continous periodic signal which has

$T = 1/f_m$

And, as it is periodic.

$$e^{j\beta \sin 2\pi f_m t} = e^{j\beta \sin 2\pi f_m (t + \frac{1}{f_m})} \quad [\because T = 1/f_m]$$

* fourier series is used to find the frequency analysis of continous periodic signal. So, the exponential fourier series is given as:-

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad ; \quad \omega_0 = \frac{2\pi}{T}$$

where, $$C_n = \frac{1}{T} \int_{-T/2}^{T/2} C_n e^{-jn\omega_0 t}$$

Now,

$$\cancel{C_n = \frac{1}{(1/f_m)} \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t} \cdot e^{j}}$$

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{\cancel{j2\pi f_m t}\, jn 2\pi f_m t} \quad -(2)$$

where,

$$C_n = \frac{1}{(1/f_m)} \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t} \cdot e^{-jn 2\pi f_m t} \, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)} \, dt$$

Now, as $J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin\theta - n\theta)} \, d\theta$

So, $\theta = 2\pi f_m t$

$d\theta = 2\pi f_m \, dt$

$\to \quad t = -\frac{1}{2f_m} \Rightarrow \theta = \cancel{2}\pi \cancel{f_m} \times -\frac{1}{\cancel{2 f_m}} = -\pi$

$\to \quad t = \frac{1}{2f_m} \Rightarrow \theta = \cancel{2}\pi \cancel{f_m} \times \frac{1}{\cancel{2 f_m}} = +\pi$

changing the integral variable.

Now, it becomes

$$C_n = \cancel{f_m} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \cdot \frac{d\theta}{2\pi \cancel{f_m}}$$

$$\boxed{= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \cdot d\theta = J_n(\beta)} \quad [\because x = \beta]$$

So, $\boxed{C_n = J_n(\beta)}$

Putting this value in eq^n ②, we get :-

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn 2\pi f_m t}$$

Now, substituting this value in eq^n ① :-

$$S_{FM}(t) = A_c \, Re\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \, e^{jn 2\pi f_m t} \cdot e^{j2\pi f_c t} \right\}$$

$$= A_c \, Re\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \, e^{j2\pi(f_c + n f_m)t} \right\}$$

So, general expression of WBFM becomes :-

$$\boxed{S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t}$$

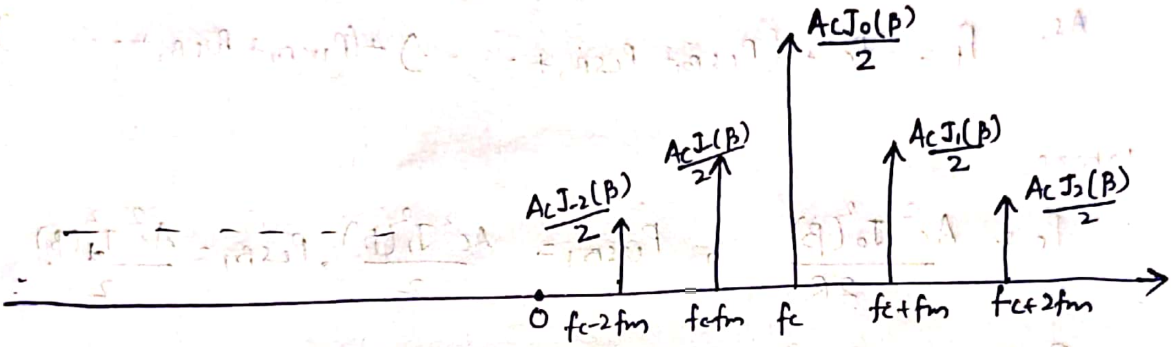
Analysis :-

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m) t$$

$$= \overset{n=0}{A_c J_0(\beta) \cos 2\pi f_c t} + \overset{n=1}{A_c J_1(\beta) \cos 2\pi (f_c + f_m) t} + \dots$$

$$+ \overset{n=-1}{A_c J_{-1}(\beta) \cos 2\pi (f_c - f_m) t} + \overset{n=+2}{A_c J_{+2}(\beta) \cos 2\pi (f_c + 2f_m) t}$$

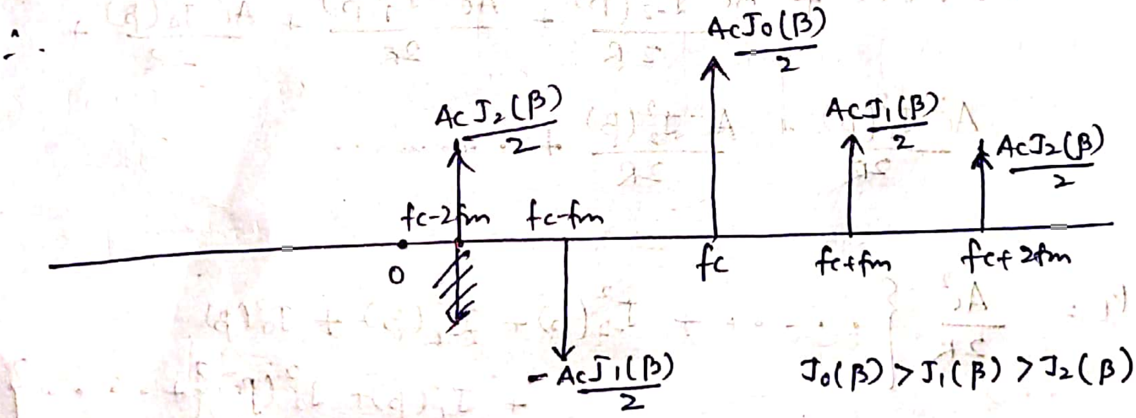
$$+ \overset{n=-2}{A_c J_{-2}(\beta) \cos 2\pi (f_c - 2f_m) t} + \dots$$



But as $J_{-1}(\beta) = -J_1(\beta)$ [from property 2]

$J_{-2}(\beta) = J_2(\beta)$

$\therefore$



$J_0(\beta) > J_1(\beta) > J_2(\beta)$

Conclusion :-

1. WBFM consists of carrier frequency. Infinite no. of LSB's and infinite no. of USB's
2. Actual Bandwidth of WBFM is ∞.

3. For WBFM, strength of higher order sidebands go on decreasing and finally becomes zero.

4. For WBFM, lower order sidebands are said to be significant sidebands and higher order sidebands are insignificant.

Power of WBFM :-

As, $P_t = P_c + (P_{LSB_1} + P_{LSB_2} + \cdots) + (P_{USB_1} + P_{USB_2} + \cdots)$

where,

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R}, \quad P_{USB_1} = \frac{A_c^2 J_1^2(\beta)}{2}, \quad P_{LSB_1} = \frac{A_c^2 J_{-1}^2(\beta)}{2}$$

$$P_{USB_2} = \frac{A_c^2 J_2^2(\beta)}{2}, \quad P_{LSB_2} = \frac{A_c^2 J_{-2}^2(\beta)}{2}$$

$$P_t = \cdots\cdots + \frac{A_c^2 J_{-2}^2(\beta)}{2R} + \frac{A_c^2 J_{-1}^2(\beta)}{2R} + \frac{A_c^2 J_0^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + \cdots\cdots$$

$$P_t = \frac{A_c^2}{2R}\left\{\cdots\cdots + J_{-2}^2(\beta) + J_{-1}^2(\beta) + J_0^2(\beta) + J_1^2(\beta) + J_2^2(\beta) + \cdots\right\}$$

$$P_t = \frac{A_c^2}{2R}\sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

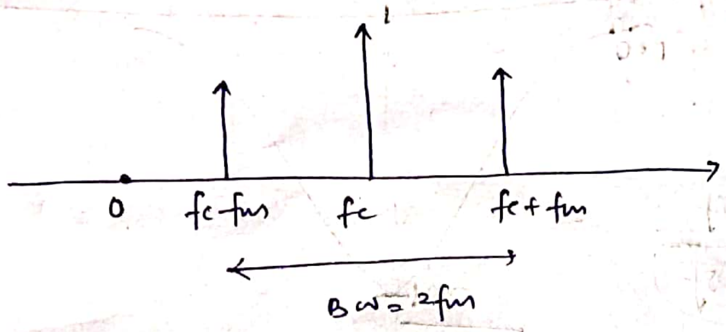
$$\therefore \boxed{P_t = \frac{A_c^2}{2R}} \quad \left[\because \sum_{n=-\infty}^{\infty} J_n^2(x) = 1\right]$$

i.e for WBFM, the power of carrier before modulation and after modulation remains same.

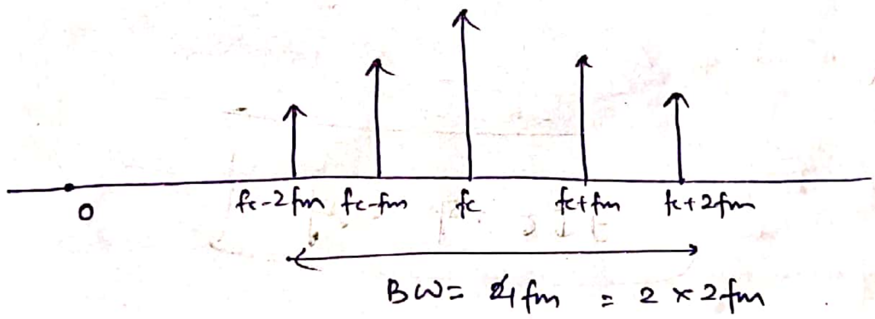
Practical Bandwidth of WBFM (Carson's Rule):-

- The actual bandwidth of WBFM is infinite. But, for transmission of signal, it should be bandlimited by retaining only significant sidebands and ~~eliminita~~ eliminating insignificant sidebands.

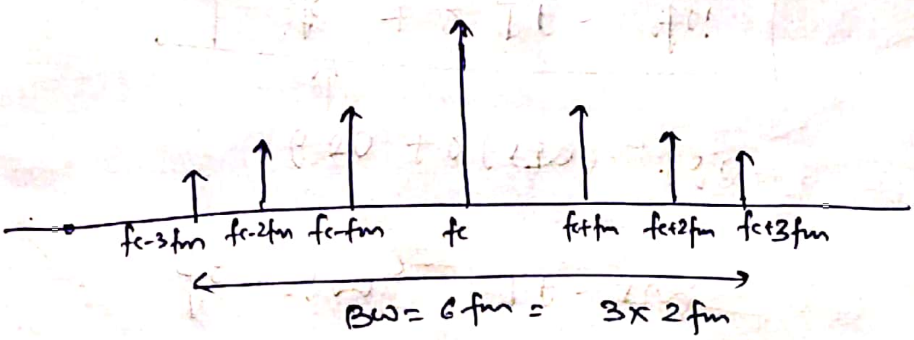
Case1:- (WBFM consist of significant sidebands upto 1st order)



Case 2:- (upto 2nd order)



Case 3:- (upto 3rd order):



Carson's Rule :-

A/c to Carson's rule, WBFM consists of significant sidebands upto order $(\beta+1)$, where ~~need~~ β is modulation index.

So, $\boxed{BW = (\beta+1) \times 2f_m}$

$$\boxed{BW = \left(\frac{\Delta f}{f_m} + 1\right) 2f_m}$$

$\Rightarrow$ $BW = 2\Delta f + 2f_m$

$\therefore$ $\boxed{BW = 2(\Delta f + f_m)}$