

Example on Curve fitting

EE (IVth Sem) Module - 9

Ex: 1 By the method of least squares find the best fitting straight line to the data given below.

x	5	10	15	20	25
y	15	19	23	26	30

Soln: $\rightarrow$ the equation of straight line is $y = ax + b$ — (1)
which satisfies the given tabular entry, now we have to find the values of constants a and b in eqⁿ (1) for this

normal equations are

$$\sum y = a \sum x + 5b \quad \text{--- (A)}$$

$$\sum xy = a \sum x^2 + b \sum x \quad \text{--- (B)}$$

Now

x	y	xy	x ²
5	15	75	25
10	19	190	100
15	23	345	225
20	26	520	400
25	30	750	625
75	114	1885	1375

Now

$$114 = a75 + 5b \quad \text{--- (A)}$$

$$1885 = 1375a + 75b \quad \text{--- (B)}$$

by solving (A) & (B) $a = 0.7$ $b = 12.3$

$$y = 0.7x + 12.3 \quad \text{Ans}$$

2nd Alternative method $\rightarrow$ By this method we can decrease the large value of data into simple least number value.

It means if entry of x and y shows large number, then for easy calculation we decrease the number as

$$\text{Put } X = \frac{x - A}{h} \quad \text{and } Y = \frac{y - B}{h}$$

where A is the middle value (or assumed mean) and h is the length of data.

Similarly in B also

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(3)

EX-22 Fit a Second degree Parabola to the following data taking y as dependent variable.

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Soln! - 5 is the middle value in x -entry
and average of y -entry is $\Sigma y = 52 = 6.55 \rightarrow \leftarrow 7 \rightarrow$ Present in data

thus $X = x - 5$ and $Y = (y - 7)$
c1)

Let $y = ax^2 + bx + c \rightarrow$ equation of Parabola - (1)
normal eqn of (1) is

$$\begin{aligned} \Sigma y &= a \Sigma x^2 + b \Sigma x + 9c - \\ \Sigma xy &= a \Sigma x^3 + b \Sigma x^2 + c \Sigma x \\ \Sigma x^2 y &= a \Sigma x^4 + b \Sigma x^3 + c \Sigma x^2 \end{aligned} \quad (A)$$

Now	X	Y	X^2	X^3	X^4	XY	X^2Y
	-4	-5	16	-64	256	20	-80
	-3	-1	9	-27	81	3	-9
	-2	0	4	-8	16	0	0
	-1	1	1	-1	1	-1	1
	0	3	0	0	0	0	0
	1	4	1	1	1	4	4
	2	4	4	8	16	8	16
	3	3	9	27	81	9	27
	4	2	16	64	256	8	32
	5	11	60	312	1562	55	275

$$\Rightarrow 708a + 60c = -9$$

$$60b = 51$$

$$60a + 9c = 11$$

Solving above

$$b = 0.85, a = -0.2673,$$

$$c = 3.0042$$

Hence, the eqn is

$$y = -0.2673x^2 + 0.85x + 3.0042$$

$$(y-7) = -0.2673(x-5)^2 + 0.85(x-5) + 3.0042$$

$$\Rightarrow y = -0.2673x^2 + 3.533x - 0.9283$$

(9)

Ex-3 From the table given below, find the best values of a and b in the law $y = ae^{bx}$ by the method of least squares.

x	0	5	8	12	20
y	3.0	1.5	1.0	0.55	0.18

Soln:- $y = ae^{bx}$ — (5)
 $\log_{10} y = \log_{10} a + bx \log_{10} e$

i.e. $y = A + Bx$ — (1) $b \log_{10} e = B$ $\log_{10} y = y$
 $\log_{10} a = A$

Normal eqn are

$$\begin{cases} \sum y = 5A + B \sum x \\ \sum xy = A \sum x + B \sum x^2 \end{cases} \rightarrow (A)$$

Now	x	y	$y = \log_{10} y$	xy	x^2	Put in (A)
	0	3.0	$\log_{10} 3.0 = 0.4771$	0	0	$5A + 45B = -0.3511$
	5	1.5	$\log_{10} 1.5 = 0.1761$	0.8805	25	$45A + 633B = -17.1887$
	8	1.0	$\log_{10} 1.0 = 0$	0	64	$\Rightarrow A = 0.4815$
	12	0.55	$\log_{10} 0.55 = -0.2596$	-3.1152	144	$= \log_{10} a$
	20	0.18	$\log_{10} 0.18 = -0.7447$	-14.894	400	Now $\Rightarrow a = 10^{0.4815}$
			-0.3511 $= \sum y$	-17.1887 $= \sum xy$	633	$a = 3.0304$

$$b \log_{10} e = -0.0613$$

$$\text{And } B = -0.0613$$

$$b = -0.0613 \times \log_{10} e$$

$$= -0.1411$$

Now Put in given eqn (5)

$$y = 3.0304 e^{-0.1411x}$$

(5)

Assignment

(i) By the method of least squares, find the straight line that best fits the following data.

x	1	2	3	4	5
y	14	27	40	55	68

(ii) Fit a parabola $y = a + bx + cx^2$ to the following data.

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

(iii) Fit a least square geometric curve $y = ax^b$ to the following data.

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(iv) Using the method of least squares, fit the curve $y = ax^2 + b/x$ to the following data.

x	1	2	3	4
y	-1.51	0.99	8.88	7.66