Mathematics III

Examples on Gauss-forward interpolation and Backward interpolation (EE + ECE / 2nd semester) Module – 2

with assignment

Ex→1 Apply Gauss's forward formula to obtain f(x) at x = 3.5 from the table below.

x	2	3	4	5
f(x)	2.626	3.454	4.784	6.986

Soln:- Let $x = x_0 + rh$ → 1 (difference in x)

3.5 ↙ ↘ origin

Now $r = \frac{x - x_0}{h} = \frac{3.5 - 3}{1} = 0.5$

Now by central difference table.

[Gauss forward interpolation → start with]

x	y	Δy First diff	$\Delta^2 y$	$\Delta^3 y$
$x_{-1}=2$	$2.626 = y_{-1}$	$\Delta y_{-1} = 0.828$		
$x_0 = 3$	$3.454 = y_0$		$\Delta^2 y_{-1} = 0.502$	
		$\Delta y_0 = 1.330$		$\Delta^3 y_{-1} = 0.37$
$x_1 = 4$	$4.784 = y_1$		$\Delta^2 y_0 = 0.872$	
$x_2 = 5$	$6.986 = y_2$	$\Delta y_1 = 2.202$		

$\Delta y_{-1} = y_0 - y_{-1} = 0.828$, $\Delta y_0 =$

Now Gauss forward interpolation. $f(3.5) = y(3.5) =$

2 Pair

$$= r_0 y_0 + r_1 \Delta y_0 + r_2 \Delta^2 y_{-1} + (r+1)_3 \Delta^3 y_{-1}$$

(r increase → , r increase → 3)

$$= 1 \times 3.454 + r(4.784 - 3.454) + \frac{r(r-1)}{\lfloor 2} \times 0.502 + \frac{\lfloor r+1}{\lfloor 3 \lfloor r+1-3} \times 0.37$$

$$= 3.454 + 0.5 \times 1.330 + \frac{0.5(0.5-1)}{2} \times 0.502 + \frac{(r+1)r(r-1)\lfloor r-2}{\lfloor 3 \lfloor r-2} \times 0.37$$

$$\Rightarrow y(3.5) = 3.454 + 0.665 - 0.06275 + \frac{(0.5+1)(0.5)(0.5-1)\times 0.37}{3\times 2\times 1}$$

$$\Rightarrow y(3.5) = 4.033125$$

Ex⇒2 Find the value of $\cos 51°42'$ by using Gauss's backward interpolation formula from the table given below.

x	$50°$	$51°$	$52°$	$53°$	$54°$
$y=\cos x$	0.6428	0.6293	0.6157	0.6018	0.5878

Soln:- $\rightarrow 52°$ origin

$$x = x_0 + rh$$

$$51°42' = 52° + r\times 1°$$

$$\Rightarrow \frac{51°42' - 52°}{1°} = r \Rightarrow \frac{(51\times 60)' + 42' - (52\times 60)'}{(60)'} = r$$

$$\Rightarrow \frac{60'(51-52)}{60'} = (-0.3)' = r$$

Now by Central difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$50°$	$y_{-2}=0.6428$	$\Delta y_{-2} = -0.0135$			
$51°$	$y_{-1}=0.6293$	$\Delta y_{-1} = -0.0136$	$\Delta^2 y_{-2} = -0.0001$	$\Delta^3 y_{-2} = -0.0002$	$\Delta^4 y_{-2} = 0.0004$
$52°$	$y_0=0.6157$	$\Delta y_0 = -0.0139$	$\Delta^2 y_{-1} = -0.0003$	$\Delta^3 y_{-1} = 0.0002$	
$53°$	$y_1=0.6018$	$\Delta y_1 = -0.0140$	$\Delta^2 y_0 = -0.0001$		
$54°$	$y_2=0.5878$				

use → Backward arrow

Now Gauss Backward interpolation

$\{(51\times 60+42)'\}$ = single ' single

$$= {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + \underbrace{(r+1)}_{\text{take with next}} {}^{r+1}C_2\, \Delta^2 y_{-1} + \underbrace{(r+1)}_{\text{take with next}} {}^{r+1}C_3\, \Delta^3 y_{-2} + \underbrace{(r+2)}_{\text{take with next}} {}^{r+2}C_4\, \Delta^4 y_{-2} =$$

Note
$nC_0 = 1$
$nC_1 = n$

(3) (EE Mathath II)

18 Present

NOCO

$$f(51^\circ 42') = 1\times 0.6157 + r\times -0.0136 + \frac{\lfloor r+1}{\lfloor 2 \lfloor r+1-2} \times -0.0003$$

$$+ \frac{\lfloor r+1}{\lfloor 3 \lfloor r+1-3} \times -0.0002 + \frac{\lfloor r+2}{\lfloor 4 \lfloor r+2-4} \times 0.0004$$

$$\Rightarrow f(51^\circ 42') = 1\ 0.6157 + (-0.3)(-0.0136) + \frac{\lfloor r+1}{\lfloor 2 \lfloor r-1} \times -0.0003$$

$$+ \frac{\lfloor r+1}{\lfloor 3 \lfloor r-2} \times -0.0002 + \frac{\lfloor r+2}{\lfloor 4 \lfloor r-2} \times 0.0004$$

$$= 0.6157 + 0.3\times 0.0136 + \frac{(r+1)r\lfloor r-1}{2\times 1 \lfloor r-1} \times -0.0003$$

$$+ \frac{(r+1)r(r-1)\lfloor r-2}{3\times 2\times 1 \lfloor r-2} \times -0.0002 + \frac{(r+2)(r+1)r(r-1)\times \lfloor r-2}{4\times 3\times 2\times 1 \lfloor r-2} \times 0.000$$

$$= 0.6157 + 0.3\times 0.0136 + \frac{(-0.3+1)(-0.3)}{2\times 1} \times -0.0003$$

$$+ \frac{(-0.3+1)(-0.3)(-0.3-1)}{3\times 2\times 1} \times -0.0002$$

$$+ \frac{(-0.3+2)(-0.3+1)(-0.3)(-0.3-1)}{4\times 3\times 2\times 1} \times 0.0004$$

$$\boxed{\cos 51^\circ 42' = 0.6198}$$ MS//

Assignment

(1) By Gauss-forward formula Find $e^{-1.7425}$ if $y = e^{-x}$ values given in table below.

x	1.72	1.73	1.74	1.75	1.76
$y = e^{-x}$	0.17907	0.17728	0.17552	0.17377	0.17204

[Take $x_0 = 1.74$]