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Gauss Backward interpolation Module – Branch ECE + EE (IInd Sem)

Proof:– By Newton's forward interpolation formula at $x = x_0$, we have.

$$y(x) = f(x) = f(x_0 + rh) = f(x_0) + r\Delta f(x_0) + \frac{r(r-1)}{\lfloor 2} \Delta^2 f(x_0) + \frac{r(r-1)(r-2)}{\lfloor 3} \Delta^3 f(x_0) + \cdots$$

$$= y_0 + r\,\Delta y_0 + \frac{r(r-1)}{\lfloor 2} \Delta^2 y_0 + \frac{r(r-1)(r-2)}{\lfloor 3} \Delta^3 y_0 + \cdots \quad (1)$$

where $r = \dfrac{x - x_0}{h}$

Since $\Delta y_0 = \Delta E y_{-1} = \Delta(1+\Delta) y_{-1} = \Delta y_{-1} + \Delta^2 y_{-1}$

i.e. $\Delta y_0 = \Delta y_{-1} + \Delta^2 y_{-1}$

$\Delta^2 y_0 = \Delta^2 y_{-1} + \Delta^3 y_{-1}$

$\Delta^3 y_0 = \Delta^3 y_{-1} + \Delta^4 y_{-1}$

$\Delta^3 y_{-1} = \Delta^3 y_{-2} + \Delta^4 y_{-2}$ etc

For this

we should have to understand the terms which are used in formula.

x	y	First diff	2nd dif	3rd dif	fourth diff	5th diff	6th
$a-3h$	y_{-3}						
		~~Δy_{-3}~~ Δy_{-3}					
$a-2h$	y_{-2}		$\Delta^2 y_{-3}$				
		Δy_{-2}		$\Delta^3 y_{-3}$			
$a-h$	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$	
a	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$	
$a+h$	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		
		Δy_1		$\Delta^3 y_0$			
$a+2h$	y_2		$\Delta^2 y_1$				
		Δy_2					
$a+3h$	y_3						

Only arrow entry are used in formula

According to above entry substituting the values of Δy_0, $\Delta^2 y_0$ and $\Delta^3 y_0$ ---- in terms of Δy_{-1}, $\Delta^2 y_{-1}$, $\Delta^3 y_{-2}$ in (1)

~~Now equal~~

Now eqn (1) becomes

$$y_0 + r(\Delta y_{-1} + \Delta^2 y_{-1}) + \frac{r(r-1)}{\lfloor 2}(\Delta^2 y_{-1} + \Delta^3 y_{-1}) + \frac{r(r-1)(r-2)}{\lfloor 3}(\Delta^3 y_{-1} + \Delta^4 y_{-1})$$
$$+ \frac{r(r-1)(r-2)(r-3)}{\lfloor 4}(\Delta^4 y_{-1} + \Delta^5 y_{-1}) + \cdots \quad -(1)$$

but Only Δy_{-1} and $\Delta^2 y_{-1}$ entry used according to formula and next entry is $\Delta^3 y_{-2}$, so that (1) becomes

$$ {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + ({}^{r}C_1 + {}^{r}C_2)\, \Delta^2 y_{-1} + ({}^{r}C_2 + {}^{r}C_3)\, \Delta^3 y_{-1}$$
$$+ ({}^{r}C_3 + {}^{r}C_4)\, \Delta^4 y_{-1} + \cdots$$

$$= {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + {}^{(r+1)}C_2\, \Delta^2 y_{-1} + {}^{(r+1)}C_3\, \Delta^3 y_{-1} + {}^{(r+1)}C_4\, \Delta^4 y_{-1} + \cdots$$

$$= {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + {}^{(r+1)}C_2\, \Delta^2 y_{-1} + {}^{(r+1)}C_3 \left[\Delta^3 y_{-2} + \Delta^4 y_{-2}\right]$$
$$+ {}^{(r+1)}C_4 \left[\Delta^4 y_{-2} + \Delta^5 y_{-2}\right] + \cdots$$

$$= {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + {}^{(r+1)}C_2\, \Delta^2 y_{-1} + {}^{(r+1)}C_3\, \Delta^3 y_{-2} + \left[{}^{(r+1)}C_3 + {}^{(r+1)}C_4\right]$$
$$\Delta^4 y_{-2} + \cdots$$

$$= {}^{r}C_0\, y_0 + {}^{r}C_1\, \Delta y_{-1} + {}^{(r+1)}C_2\, \Delta^2 y_{-1} + {}^{(r+1)}C_3\, \Delta^3 y_{-2} + {}^{(r+2)}C_4\, \Delta^4 y_{-2} + \cdots$$

Now formula is remember as

Coefficient	${}^{r}C_0$	${}^{r}C_1$	${}^{(r+1)}C_2$	${}^{(r+1)}C_3$	${}^{(r+2)}C_4$	${}^{(r+2)}C_5$
Differences	y_0	$\rightarrow \Delta y_{-1}$	$\searrow \Delta^2 y_{-1}$	$\nearrow \Delta^3 y_{-2}$	$\searrow \Delta^4 y_{-2}$	$\nearrow \Delta^5 y_{-3}$