Dynamics of fluid flow

In the last chapter i.e, kinematics of fluid flow we studied velocity and accⁿ without taking into consideration of any forces. In this chapter we will study the forces causing flow of fluid. Actually dynamics refers to the study of motion of fluid considering forces.

Generally the forces acting in fluid are pressure force (F_p), gravity force (F_g) and viscous force (F_v).

Formula:

F_p = Pressure × Area = PA

F_g = mass × Acceleration due to gravity = $\rho V g$

$F_v = \mu A \frac{V}{y}$

In navier stoke analysis all these three forces are taken into consideration. If viscous forces are neglected then the forces acting on the fluid element are pressure forces (F_p) and gravity force (F_g).

In Euler's analysis only pressure force and gravity force are taken into consideration.

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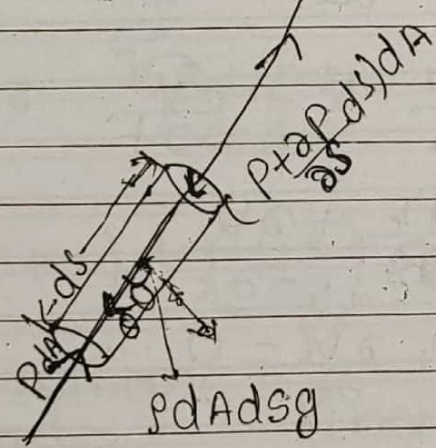
* Euler's equation of motion:

This is the equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of fluid element along a stream line.



$W_t = \rho dVg$
$= \rho dA ds g.$
$\rho = \frac{m}{V}$
$dm = \rho dV$

consider a cylindrical element of cross sectional area dA and length ds in a streamline of fluid flow. The forces acting on this element are (i) pressure force $= pdA$

(ii) pressure force $= (p + \frac{\partial p}{\partial s} ds)dA$

(iii) Gravity force or wt $W = \rho dA ds g$, as shown in the figure.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element × Accn in the dirn of S

$\therefore F_s = m \cdot a_s$

$\Rightarrow pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho\, dA\, ds\, g\cos\theta = \rho\, dA\, ds \cdot a_s$ —— (i)

Now, $a_s = \frac{dV}{dt}$

$= \frac{\partial V}{\partial s} \cdot \frac{\partial s}{\partial t} + \frac{\partial V}{\partial t}$

$= \frac{\partial V}{\partial s} \cdot V + \frac{\partial V}{\partial t}$

$= V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$

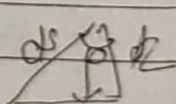
For steady flow, $\frac{\partial V}{\partial t} = 0$

$a_s = \frac{V dV}{ds}$

from (i),

$\cancel{pdA} - \cancel{pdA} - \frac{\partial p}{\partial s} ds\, dA - \rho\, dA\, ds\, g\cos\theta = \rho\, dA\, \cancel{ds}\, V dV$

$\Rightarrow -\frac{\partial p}{\partial s} ds\, \cancel{dA} - \rho\, \cancel{dA}\, ds\, g\cos\theta = \rho\, \cancel{dA}\, V dV\, \cancel{ds}$



$\cos\theta = \frac{dz}{ds}$

$= -\frac{\partial p}{\cancel{\partial s}} \cancel{ds} - \rho\, \cancel{ds}\, g \frac{dz}{\cancel{ds}} = \rho V dV$

$\Rightarrow -\partial p - \rho g\, dz = \rho V dV$

$= -dp - \rho g\, dz = \rho V dV$

$\Rightarrow \rho v\,dv = -dP - \rho g\,dz$

$\Rightarrow \rho v\,dv + dP + \rho g\,dz = 0$

$\Rightarrow dP + \rho v\,dv + \rho g\,dz = 0$

Dividing by ρ

$\frac{dP}{\rho} + \frac{\cancel{\rho} v\,dv}{\cancel{\rho}} + \frac{\cancel{\rho} g\,dz}{\cancel{\rho}} = 0$

$$\boxed{\frac{dP}{\rho} + v\,dv + g\,dz = 0}$$

This is the Euler's equation of motion.

* Assumptions:

(i) The viscosity of fluid is neglected, i.e, ideal fluid.

(ii) The flow is steady

(iii) The flow is incompressible

(iv) The flow is irrotational.

* Bernoulli's equation.

From Euler's equation

$\frac{dP}{\rho} + g\,dz + v\,dv = 0$

Integrating both sides we have,

$\int \frac{dP}{\rho} + g\int dz + \int v\,dv = \int 0$

$$\boxed{\frac{P}{\rho} + \underbrace{g.z}_{\text{Potential}} + \underbrace{\frac{v^2}{2}}_{K.E} = \text{constant}}$$

Pressure energy

Dividing by g, we have,

$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \frac{\text{constant}}{g}$

$\left(m \cdot \frac{P}{\rho}\right)$

$$\frac{\text{energy}}{\text{mass}} \rightarrow \boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{constant}}$$

This is the Bernoulli's equation.

In the above equation (2) each term represents energy of the fluid per unit mass.

$$\frac{\text{Energy}}{\text{mass}} \boxed{\frac{P}{\rho} + \frac{v^2}{2} + gz = \cancel{0} \text{ constant}}$$

$\frac{\text{Pressure energy}}{\text{mass}}$ $\frac{K.E}{\text{mass}}$ $\frac{\text{Potential energy}}{\text{mass}}$

$$\frac{Pm}{\rho} + \frac{mv^2}{2} + mgz = \text{constant}$$

Pressure energy, K.E, Potential energy

Above expression shows that the sum of pressure energy, K.E and potential energy is constant

Statement :→ It states that for a steady, nonviscous and incompressible fluid the sum of pressure energy, potential energy and K.E will remain constant along a stream line.

It means bernouli eqn is nothing but energy conservation principle.

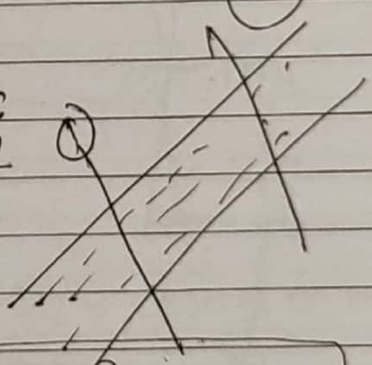
$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

$\frac{P}{\rho g}$ → pressure head

$\frac{v^2}{2g}$ → K.E kinetic head

z → potential head

$$\underbrace{\left(\frac{P}{\rho g} + z\right)}_{\text{Piezometric head}} + \underbrace{\left(\frac{v^2}{2g}\right)}_{\text{kinetic head}} = \text{constant}$$



$$\boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2}$$

$$\boxed{A_1 V_1 = A_2 V_2}$$

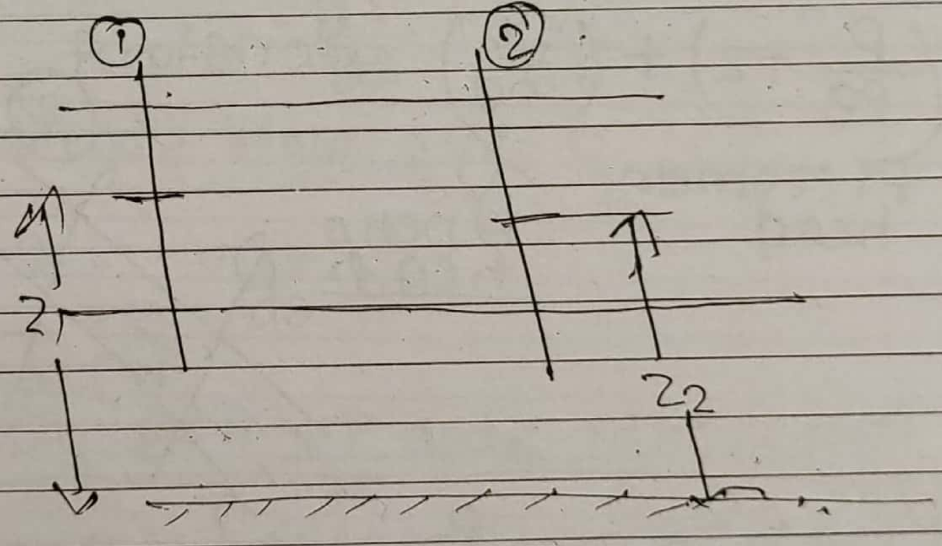
$$\boxed{\left(\frac{P_1}{\rho g} + z_1\right) + \frac{v_1^2}{2g} = \left(\frac{P_2}{\rho g} + z_2\right) + \frac{v_2^2}{2g}}$$

$$= \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{V_2^2 - V_1^2}{2g} = h$$ (Piezo metric head difference)

$$\Rightarrow \frac{V_2^2 - V_1^2}{2g} = h$$

$$\Rightarrow \boxed{V_2^2 - V_1^2 = 2gh}$$

<u>for horizontal pipe:</u>

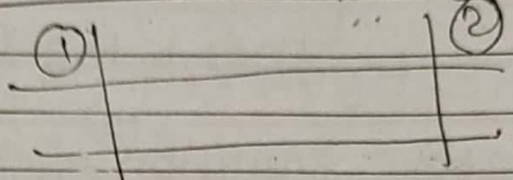


here $z_1 = z_2$
so Bernoullis equation becomes

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}}$$

Bernoulis equation bor real bluid.



$E_1 = E_2$

$E_2 +$

=

The Bernoulis equation was derived by taking the assumption that ~~bluid in~~ ~~[illegible]~~ is inviscit (non viscous) and therebore brictionless. But bor real bluid there will be some value ob viscosity which obbers resistance to blow. Thus there are always some looses in bluid blow at hence in the application ob Bernoulis equation These looses have to be taken into consideration. Thus Bernoulis eqn bor real bluid b/w Pt 1 and 2 is given as

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L}$$

where h_L is the loss ob energy b/w Pt 1 and 2