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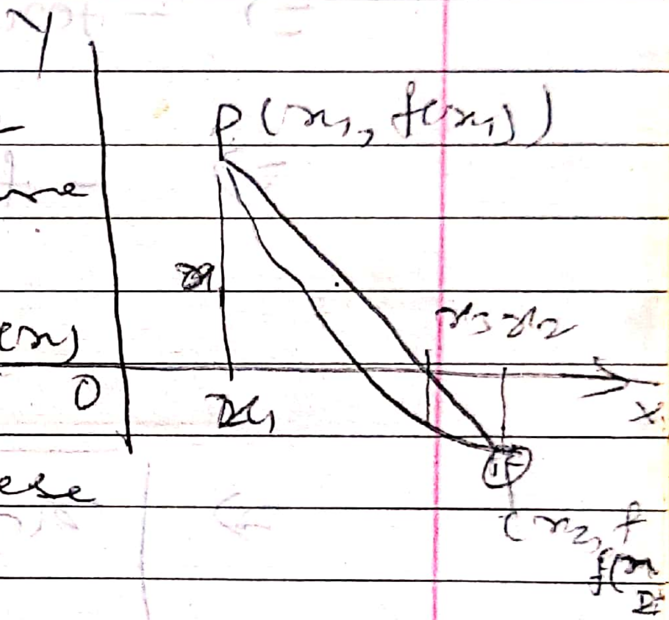
Regula falsi method

(or method of false position)

↓

is oldest method to find the approximate numerical value of a real root of an equation $f(x)=0$

Here we select two points x_1 and x_2 s.t. $f(x_1)$ & $f(x_2)$ are of opposite signs.
i.e The curve $y=f(x)$ crosses the x-axis of x between these points.



∴ Root of the equation $f(x)=0$ lies between x_1 and x_2

$\Rightarrow \boxed{f(x_1) \cdot f(x_2) < 0}$

We shall derive a formula to compute the root

i) The equation of chord PQ is given by

$$y - f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$$

The chord cuts x-axis, $y=0$, let $x=x_3$

$\therefore\ -f(x_1) = \frac{f(x_2)-f(x_1)}{x_2-x_1}\cdot(x_3-x_1)$

$\Rightarrow\ -f(x_1)\cdot(x_2-x_1) = (x_3-x_1)[f(x_2)-f(x_1)]$

$\Rightarrow\ \frac{-f(x_1)(x_2-x_1)}{f(x_2)-f(x_1)} = x_3-x_1$

$$\Rightarrow\ \boxed{x_3 = x_1 - \frac{(x_2-x_1)}{f(x_2)-f(x_1)}\times f(x_1) \quad -(1)}$$

(1) gives an approximation of the root. To find the better approximation, we evaluate $f(x_3)$. $f(x_1)$ & $f(x_3)$ are opposite sign, then the root lies bet^n x_1 & x_3. Replacing x_2 by x_3 in (1) we get new approximation x_4 to the root.

The process is repeated until the roots is obtained to the desired accuracy

~~2~~ 3

e.g. Find a real root of the equation $x^3 - 9x + 1 = 0$ by the method of false position.

$$f(x) = x^3 - 9x + 1$$

$x = 1$

$$f(1) = 1 - 9 + 1 = 2 - 9 = -7$$
$= (-)ve$

$x = 2$

$$f(2) = 8 - 9\times2 + 1 = 9 - 18 = -9$$
$= (-)ve$

$x = 3$

$$f(3) = 27 - 9\times3 + 1 = 1 = (+)ve$$

∴ Root lies between 2 & 3

∴ From method of false position

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \times f(x_1)$$

$$= 2 - \frac{3-2}{1+9} \times (-9)$$

$$= 2 + \frac{1}{10} \times 9$$

$$= 2 + 0.9 = 2.9$$

(4)

Now $f(x_3) = f(2.9)$

$= (2.9)^3 - 9(2.9) + 1$

$= -0.711$

Obviously $f(x_2) \cdot f(x_3) < 0$

$\Rightarrow$ ~~Root lies between 2.9 & 3~~

Now, take $x_1 = 2.9$, $x_2 = 3$,

~~$f(x_1) = -0.711$, $f(x_2) = 1$~~

$\therefore x_4 = 2.9 - \frac{3-2.9}{1+0.711} \times (-0.711)$

$= 2.9416$

Now, $f(x_4) = f(2.9416)$

$= (2.9416)^3 - 9(2.9416) + 1$

$= -0.0207$

Obviously $f(x_2) \cdot f(x_4) < 0$

$\therefore$ Root lies between 2.9416 & 3

Take $x_1 = 2.9416$, $x_2 = 3$

$f(x_1) = -0.0207$, $f(x_2) = 1$

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$\therefore x_5 = 2.9416 - \frac{0.0584}{1.0207} \times (-0.0207)$

$= 2.9428$

$-\frac{f(x_1)(x_2-x_1)}{f(x_2)-f(x_1)}$

Now $f(x_5) = -.0003$, proceeding as above

$x_6 = 2.9428 - \frac{0.0572}{1.0003} \times (-0.0003)$

$= 2.9428$

$\Rightarrow f(x_1)(x_3-x_1) - f(x_1)(x_3-x_1) = f(x_1)(x_2-x_1)$

$\Rightarrow f(x_1)[x_3-x_1-x_2+x_1] = f(x_2)(x_3-x_1)$

$= \frac{f(x_2)}{f(x_1)}(x_3-x_1)$

$x_3 - x_2 = \frac{f(x_2)}{f(x_1)}$

$x_3 = x_2 + \frac{f(x_2)}{f(x_1)}$

~~The root is 2.942 correct~~ to three decimal places

(2) Find the real root of the equation $x \log_{10} x - 1.2 = 0$ by the method of false position correct to four decimal places

(6)

Let $f(x) = x\log_{10}x - 1.2$

$x = 2$

$\Rightarrow f(2) = 2\log_{10}2 - 1.2$

$= -0.59794$

$\log_{10}3 = 0.477$ $\quad f(3) = 3\log_{10}3 - 1.2$

$= 0.23136$

$\therefore$ one root lies between 2 & 3

Let $x_1 = 2$, $x_2 = 3$

$f(x_1) = -0.59794$

$f(x_2) = +0.23136$

From method of false position

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \times f(x_1)$$

$$= 2 - \frac{3-2}{0.23136 + 0.59794} \times (-0.59794)$$

$= 2.72102$

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$\therefore f(x_3) = f(2.72102)$

$= (2.72102) \log_{10} 2.72102 - 1.2$

$= -.01709$

$\Rightarrow f(2.72102) \cdot f(3) < 0$

Root lies between x_3 & 3

Take $x_1 = 2.72102$

$x_2 = 3$

$f(x_1) = -.01709$

$f(x_2) = 0.23136$

$\therefore x_4 = 2.72102 - \frac{(3-2.72102)}{0.24845} \times (-0.01709)$

$= 2.74021$

Now, $f(x_4) = -0.00038$

$\therefore f(x_2) \cdot f(x_4) < 0$

∴ Root lies between 2.74021 & 3

Take $x_1 = 2.74021$, $x_2 = 3$

$f(x_1) = -0.00038$

$f(x_2) = 0.23136$

$$\therefore x_5 = 2.74021 - \frac{0.25979}{0.23174} \times (-0.00038)$$

$$= 2.74064$$

Now, $f(x_5) = -0.00001$

Proceeding as above

$$x_6 = 2.74064 - \frac{0.25936}{0.23137} \times (-0.00001)$$

$$= 2.74065$$

∴ The root correct to decimal places is 2.7406

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(3) Apply false position method to solve the equation

$3x - \cos x - 1 = 0$

Sol^n:- Let $f(x) = 3x - \cos x - 1$

Here $f(.60) = -0.02534$

$f(.61) = 0.01035$

One root lies between .60 & .61

Taking $x_1 = .60$, $x_2 = .61$

$f(x_1) = -0.02534$

$f(x_2) = 0.01035$

∴ By the method of false position

$$x_3 = x_1 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_1)$$

$$= 0.60 - \frac{.01}{0.03569} \times (-0.02534)$$

$= 0.60710$

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$\therefore f(x_3) = f(.60710)$

$= -0.00001$

Taking $x_1 = .60710$, $x_2 = .61$

$f(x_1) = -.00001$

$f(x_2) = 0.01035$

$\therefore x_4 = .60710 - \dfrac{0.00290}{0.01036} \times (-.00001)$

$= .60710$

$\therefore$ Root correct to four decimal places is 0.6071