

(1)

classmate

Date _____

Page _____

Fourier Transforms (Generalise)

form of Fourier series

We know that Fourier series is defined as in periodic $f(x)$ in the interval $(-1, 1)$

$$\text{i.e. } f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{a_0}{2} + [a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x$$

$$+ \dots + a_n \cos nx]$$

$$+ [b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x$$

$$+ \dots + b_n \sin nx]$$

Where $a_0 = \frac{1}{1} \int_{-1}^1 f(x) dx$

$$a_n = \frac{1}{1} \int_{-1}^1 f(x) \cos nx dx$$

$$b_n = \frac{1}{1} \int_{-1}^1 f(x) \sin nx dx$$

But since $\sin nx$ & $\cos nx$ are periodic between 0 to 2π .

$\therefore a_0, a_n, b_n$ may also be expressed as

②

classmate

Date

Page

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx$$

i.e. In other words A periodic function $f(x)$ defined

in a finite interval $(-l, l)$

Can be expressed as a Fourier series

When we define the non periodic function in

$-\infty < x < \infty$, $\forall x$ can be expressed as a

Fourier integral.

i.e. A function $f(x)$ is piecewise continuous in every finite interval

and is absolutely integrable

on the x -axis, can be expressed

(3)

classmate

Date _____

Page _____

by a Fourier integral

$$f(x) = \int_0^{\infty} [A(x) \cdot \cos bx + B(x) \sin bx] dx$$

is valid ~~for~~ at all pts of
Continuity.

At a point discontinuity

x_0 , the Fourier integral
is the average of the left and
right hand limits

$$= \frac{1}{2} [f(x_0-0) + f(x_0+0)]$$

Standard results

$$1. \int_0^{\infty} e^{an} f(bn) dn = \frac{e^{an}}{a^2 + b^2} [a \cos bn - b \sin bn]$$

$$2. \int_0^{\infty} e^{-an} \cdot f(bn) dn = \frac{b}{a^2 + b^2}$$

$$3. \int_0^{\infty} e^{an} \cdot \cos bn dn = \frac{e^{an}}{a^2 + b^2} [a \cos bn + b \sin bn]$$

$$4. \int_0^{\infty} e^{-an} \cdot f(bn) dn =$$

$$4. \int_0^{\infty} e^{-an} \cdot \cos bn dn = \frac{a}{a^2 + b^2}$$

④ ⑤ $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

(6) $\int_0^{\infty} \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}} dx = \frac{1}{2} \tan \frac{a}{2}$

(7) $\int_0^{\infty} \frac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}} dx = \frac{1}{2} \sec \frac{a}{2}$

(8) $\int_0^{\infty} \frac{e^{-ax}}{x} \cdot \ln bx dx = \tan^{-1} b/c$
 $C > 0, b > 0$

(9) $\int_0^{\infty} \frac{\ln ax}{x} dx = \pi/2$ if $a > 0$

Fourier Transform

From the Fourier integral representation we get

Fourier ~~integral~~ transform

(which is complex).

Fourier cosine & sine transform

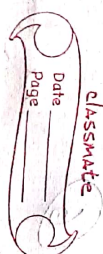
(which are real) of $f(x)$

i.e. Fourier transform of $f(x)$

The Fourier integral of $f(x)$

in the complex form is given by

5 (5)



$$f_{xy} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{-ixt} dt e^{ixr} dx$$

— (1)

(1) may be written as

$$f_{xy} = \int_{-\infty}^{\infty} f(x) \cdot e^{-ixr} dx \quad \text{--- (2)}$$

$$\text{Where } f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_{xy} \cdot e^{-ixr} dx \quad \text{--- (3)}$$

$f(x)$ defined on (2) is

known as the Fourier transform of f_{xy} .

f_{xy} defined by (2) is called the inverse Fourier transform of $f(x)$.

Here $f(x)$ & f_{xy} are

known as Fourier

transform pair which

differ in form only

on the sign of the

exponent.

⑥

The factor $\frac{1}{2\pi}$ can be

multiply the $f(\omega)$ integral (2) instead of the $F(x)$ integral (3).

otherwise the factor $\frac{1}{\sqrt{2\pi}}$

Can multiply each of the integral in (2) & (3).

Fourier transform breaks up the function into a continuous spectrum of frequencies x .

Fourier transform

method is the process of obtaining $F(x)$ for a given function $f(\omega)$.