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(5) Solⁿ of algebraic and Transcedental equation

or Itterative Solⁿ of algebraic and Transcedental eqⁿ.

To obtain an approximate numerical solution of a given eqⁿ $f(x)=0$

The following methods are used for the solⁿ of eqⁿ

(i) Bisection method (Bolzano method)

(ii) Regula falsi method (or Method of false position)

(iii) Newton - Raphson method.

(iv) Graphical Solution

Bisection method — In this method the roots of the equation $f(x)=0$ are located between two numbers a and b (say). If the function $f(x)$ is continuous between $f(a)$ and $f(b)$ are of opposite signs, then there is a least one root between a and b.

If $f(a) = (-)ve$, $f(b) = (+)ve$

then the first approximate value of the root is $\frac{a+b}{2} = x_0$ (say)

If $f(x) = 0 \Rightarrow$ Correct root is x_0

If $f(x) \neq 0$, then the root either lies between a and x_0 or x_0 and b

according as $f(x_0)$ is $+$ve or $-$ve, we again bisect the interval as above and the process is repeated until the root is obtained to the desired accuracy.

Or

Bisection method is based on the repeated application of intermediate value property. Let the function $f(x)$ be continuous between a & b. For definiteness, let, $f(a) = -$ve & $f(b) = +$ve

Then the first approximation of the root is $x_1 = \frac{a+b}{2}$

If $f(x_1) = 0 \Rightarrow x_1$ is a root of $f(x) = 0$

Otherwise If $f(x) \neq 0$,

the root lies between a & x_1 or x_1 and b, as $f(x_1)$ is $+$ve or $-$ve.

Then we bisect the interval as before and continue the process until the root is found to desired accuracy.

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Problems on Bisection method

(1) e.g. Find the real root of the equation

$$f(x) = x^3 - x - 1 = 0$$

If $x = 1$

$\Rightarrow f(1) = 1^3 - 1 - 1 = -1 = (-)\text{ve}$

$f(2) = 2^3 - 2 - 1 = 8 - 3 = 5 = (+)\text{ve}$

$\therefore$ 1st approximation $x_0 = \frac{1+2}{2} = \frac{3}{2}$

$= 1.5$

$\Rightarrow f(x_0) = (+)\text{ve}$

$\therefore$ The root lies between 1 & 1.5

$\therefore x_1 = \frac{1+1.5}{2} = 1.25$

$\Rightarrow f(x_1) = -\frac{19}{64} = (-)\text{ve}$

$\therefore$ Roots lies between 1.25 & 1.5

$x_2 = \frac{1.25 + 1.5}{2} = 1.375$

The procedure is repeated and the successive approximations are

$x_3 = 1.3125$, $x_4 = 1.34375$,

$x_5 = 1.328125$ etc.

(2) Find a real root of the eq^n $f(x) = x^3 - 2x - 5 = 0$, using bisection method in five steps.

Sol^n: — Let $f(x) = x^3 - 2x - 5$

If $x = 1$

$f(1) = 1 - 2 - 5 = 1 - 7 = -6$
$= (-)ve$

$x = 2$

$f(2) = 8 - 4 - 5 = 8 - 9 = -1$
$= (-)ve$

Now, take

$x = 3$

$f(3) = 27 - 6 - 5 = 27 - 11$
$= 16 = (+)ve$

$\therefore$ Root lies between 2 & 3

5

∴ The first approximation to the root

$x_0 = \frac{1}{2}(2+3) = \frac{5}{2} = 2.5$

Now, $f(x_0) = f(2.5) = (2.5)^3 - 2(2.5) - 5$

$= 5.625 = (+)ve$

∴ The root lies between 2 & x_0

i.e. between 2 & 2.5

Second approximation of the root

$x_1 = \frac{1}{2}(2+2.5) = 2.25$

$\Rightarrow f(x_1) = f(2.25)$

$= (2.25)^3 - 2(2.25) - 5$

$= 1.89 = (+)ve$

∴ The root lies between 2 & x_1

∴ Third approximation of the root

$x_2 = \frac{1}{2}(2+2.25)$

$= 2.125$

$\Rightarrow f(x_2) = f(2.125) = (2.125)^3 - 2(2.125) - 5$

$= 3.46 = (+)ve$

$\therefore$ Root lies between 2 & x_2

$\therefore$ fourth approximation to the root, $x_3 = \frac{1}{2}(2+2.125)$

$= 2.0625$

$\therefore f(x_3) = f(2.0625)$

$= (2.0625)^3 - 2(2.065) - 5$

$= -8.35 = (-)ve$

$\therefore$ Root lies between x_2 & x_3

$\therefore$ fifth approximation to the root

$x_4 = \frac{1}{2}(x_2 + x_3)$

$= \frac{1}{2}(2.125 + 2.0625)$

$= 2.09375 = 2.094$

correct to 3 place of decimal

—x—