16.13 Design of Piston Rod

Since a piston rod moves forward and backward in the engine cylinder, therefore it is subjected to alternate tensile and compressive forces. It is usually made of mild steel. One end of the piston rod is secured to the piston by means of tapered rod provided with nut. The other end of the piston rod is joined to crosshead by means of a cotter.

Piston rod is made of mild steel.

- The expression $\sigma_{max} = \frac{W}{A}\left[1 + \frac{e.y_c}{k^2} \sec \frac{L}{2k}\sqrt{\frac{W}{E.A}}\right]$ may also be written as follows:

$$\sigma_{max} = \frac{W}{A} + \frac{W}{A} \times \frac{e.y_c}{\frac{I}{A}} \sec \frac{L}{2k}\sqrt{\frac{W}{E \times \frac{I}{k^2}}} \qquad \ldots\left(\text{Substituting } k^2 = \frac{I}{A} \text{ and } A = \frac{I}{k^2}\right)$$

$$= \frac{W}{A} + \frac{W.e}{Z} \sec \frac{L}{2}\sqrt{\frac{W}{E.I}}$$

Scanned with CamScanner

Let p = Pressure acting on the piston,
D = Diameter of the piston,
d = Diameter of the piston rod,
W = Load acting on the piston rod,
W_{cr} = Buckling or crippling load = $W \times$ Factor of safety,
σ_t = Allowable tensile stress for the material of rod,
σ_c = Compressive yield stress,
A = Cross-sectional area of the rod,
l = Length of the rod, and
k = Least radius of gyration of the rod section.

The diameter of the piston rod is obtained as discussed below:

1. When the length of the piston rod is small *i.e.* when slenderness ratio (l / k) is less than 40, then the diameter of piston rod may be obtained by equating the load acting on the piston rod to its tensile strength, *i.e.*

$$W = \frac{\pi}{4} \times d^2 \times \sigma_t$$

or
$$\frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times d^2 \times \sigma_t$$

$$\therefore \quad d = D\sqrt{\frac{p}{\sigma_t}}$$

2. When the length of the piston rod is large, then the diameter of the piston rod is obtained by using Euler's formula or Rankine's formula. Since the piston rod is securely fastened to the piston and cross head, therefore it may be considered as fixed ends. The Euler's formula is

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

and Rankine's formula is,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2}$$

Example 16.3. *Calculate the diameter of a piston rod for a cylinder of 1.5 m diameter in which the greatest difference of steam pressure on the two sides of the piston may be assumed to be 0.2 N/mm². The rod is made of mild steel and is secured to the piston by a tapered rod and nut and to the crosshead by a cotter. Assume modulus of elasticity as 200 kN/mm² and factor of safety as 8. The length of rod may be assumed as 3 metres.*

Solution. Given : D = 1.5 m = 1500 mm ; p = 0.2 N/mm² ; E = 200 kN/mm² = 200×10^3 N/mm² ; l = 3 m = 3000 mm

We know that the load acting on the piston,

$$W = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}\,(1500)^2 \times 0.2 = 353\,475 \text{ N}$$

$\therefore$ Buckling load on the piston rod,

$$W_{cr} = W \times \text{Factor of safety} = 353\,475 \times 8 = 2.83 \times 10^6 \text{ N}$$

Since the piston rod is considered to have both ends fixed, therefore from Table 16.2, the equivalent length of the piston rod,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500 \text{ mm}$$

Scanned with CamScanner

$$= \frac{\pi}{64}(D^4 - d^4)\text{, for tubular section}$$

l = Length of the push rod, and

E = Young's modulus for the material of push rod.

If m is the factor of safety for the long columns, then the critical or crippling load on the rod is given by

$$W_{cr} = m \times W$$

Now using Euler's formula, $W_{cr} = \frac{\pi^2 EI}{L^2}$, the diameter of the push rod (D) can be obtained.

Notes: 1. Generally the diameter of the hole through the push rod is 0.8 times the diameter of push rod, *i.e.*

$$d = 0.8\,D$$

2. Since the push rods are treated as pin end columns, therefore the equivalent length of the rod (L) is equal to the actual length of the rod (l).

Example 16.4. *The maximum load on a petrol engine push rod 300 mm long is 1400 N. It is hollow having the outer diameter 1.25 times the inner diameter. Spherical seated bearings are used for the push rod. The modulus of elasticity for the material of the push rod is 210 kN/mm². Find a suitable size for the push rod, taking a factor of safety of 2.5.*

Solution. Given : l = 300 mm ; W = 1400 N ; D = 1.25 d ; E = 210 kN/mm² = 210 × 10³ N/mm² ; m = 2.5

Let d = Inner diameter of push rod in mm, and

D = Outer diameter of the push rod in mm = 1.25 d ...(Given)

∴ Moment of inertia of the push rod section,

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(1.25\,d)^4 - d^4] = 0.07\,d^4\ \text{mm}^4$$

We know that the crippling load on the push rod,

$$W_{cr} = m \times W = 2.5 \times 1400 = 3500\ \text{N}$$

Now according to Euler's formula, crippling load (W_{cr}),

$$3500 = \frac{\pi^2 E I}{L^2} = \frac{9.87 \times 210 \times 10^3 \times 0.07\,d^4}{(300)^2} = 1.6\,d^4 \quad ...(\because L = l)$$

∴ $d^4 = 3500 / 1.6 = 2188$ or $d = 6.84$ mm **Ans.**

and $D = 1.25\,d = 1.25 \times 6.84 = 8.55$ mm **Ans.**

16.15 Design of Connecting Rod

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod subjected to an axial load W may buckle with X-axis as neutral axis (*i.e.* in the plane of motion of the connecting rod) or Y-axis as neutral axis (*i.e.* in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. A connecting rod should be equally strong in buckling about either axes.

Let A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_{cr} = Crippling or buckling load,

Scanned with CamScanner

I_{xx} and I_{yy} = Moment of inertia of the section about *X*-axis and *Y*-axis respectively, and

k_{xx} and k_{yy} = Radius of gyration of the section about *X*-axis and *Y*-axis respectively.

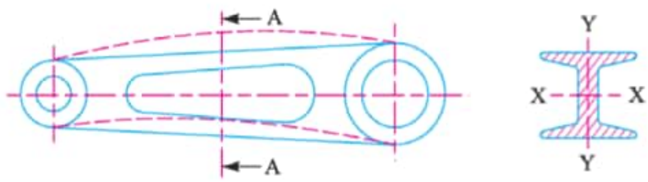

Fig. 16.6. Buckling of connecting rod.

According to Rankine's formula,

$$W_{cr} \text{ about } X\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \quad \ldots (\because \text{ For both ends hinged, } L = l)$$

and

$$W_{cr} \text{ about } Y\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2k_{yy}}\right)^2} \quad \ldots \left(\because \text{ For both ends fixed, } L = \frac{l}{2}\right)$$

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c \times A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2\,k_{yy}}\right)^2$$

$$\therefore \quad k^2_{xx} = 4\,k^2_{yy} \quad \text{or} \quad I_{xx} = 4\,I_{yy} \quad \ldots (\because I = A \times k^2)$$

This shows that the connecting rod is four times strong in buckling about *Y*-axis than about *X*-axis. If $I_{xx} > 4\,I_{yy}$, then buckling will occur about *Y*-axis and if $I_{xx} < 4\,I_{yy}$, buckling will occur about *X*-axis. In actual practice, I_{xx} is kept slightly less than $4\,I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about *X*-axis. The design will alwyas be satisfactory for buckling about *Y*-axis.

The most suitable section for the connecting rod is *I*-section with the proportions as shown in Fig. 16.7 (*a*).

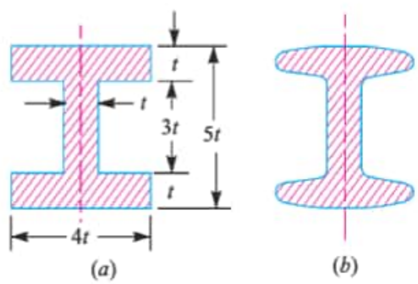

Fig. 16.7. I-section of connecting rod.

Area of the section

$$= 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

$\therefore$ Moment of inertia about *X*-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5\,t)^3 - 3\,t\,(3\,t)^3\right] = \frac{419}{12}\,t^4$$

and moment of inertia about *Y*-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}\,t \times (4\,t)^3 + \frac{1}{12}\,(3\,t)\,t^3\right] = \frac{131}{12}\,t^4$$

Scanned with CamScanner

$$\therefore \qquad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore *I*-section chosen is quite satisfactory.

Notes : 1. The *I*-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible. It can also withstand high gas pressure.

2. Sometimes a connecting rod may have rectangular section. For slow speed engines, circular sections may be used.

3. Since connecting rod is manufactured by forging, therefore the sharp corners of *I*-section are rounded off as shown in Fig. 16.7 (*b*) for easy removal of the section from the dies.

Example 16.5. *A connecting rod of length l may be considered as a strut with the ends free to turn on the crank pin and the gudgeon pin. In the directions of the axes of these pins, however, it may be considered as having fixed ends. Assuming that Euler's formula is applicable, determine the ratio of the sides of the rectangular cross-section so that the connecting rod is equally strong in both planes of buckling.*

Solution. The rectangular cross-section of the connecting rod is shown in Fig. 16.8.

Let b = Width of rectangular cross-section, and

h = Depth of rectangular cross-section.

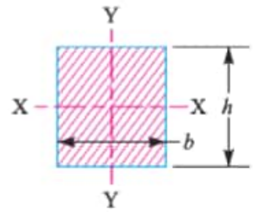

Fig. 16.8

$\therefore$ Moment of inertia about *X-X*,

$$I_{xx} = \frac{b.h^3}{12}$$

and moment of inertia about *Y-Y*,

$$I_{yy} = \frac{h.b^3}{12}$$

According to Euler's formula, buckling load,

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

$\therefore$ Buckling load about *X-X*,

$$W_{cr}\,(X\text{-axis}) = \frac{\pi^2 E I_{xx}}{l^2} \qquad \text{... } (\because L = l, \text{ for both ends free to turn})$$

and buckling load about *Y-Y*,

$$W_{cr}\,(Y\text{-axis}) = \frac{\pi^2 E I_{yy}}{(l/2)^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \text{... } (\because L = l/2, \text{ for both ends fixed})$$

In order to have the connecting rod equally strong in both the planes of buckling,

$$W_{cr}\,(X\text{-axis}) = W_{cr}\,(Y\text{-axis})$$

$$\frac{\pi^2 E I_{xx}}{l^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \text{or} \qquad I_{xx} = 4 I_{yy}$$

$$\therefore \qquad \frac{b\,h^3}{12} = \frac{4\,h\,b^3}{12} \qquad \text{or} \qquad h^2 = 4\,b^2$$

and $h^2 / b^2 = 4$ or $h / b = 2$ **Ans.**

16.16 Forces Acting on a Connecting Rod

A connecting rod is subjected to the following forces :

1. Force due to gas or steam pressure and inertia of reciprocating parts, and
2. Inertia bending forces.

We shall now derive the expressions for the forces acting on a horizontal engine, as discussed below:

Scanned with CamScanner

1. *Force due to gas or steam pressure and inertia of reciprocating parts*

Consider a connecting rod *PC* as shown in Fig. 16.9.

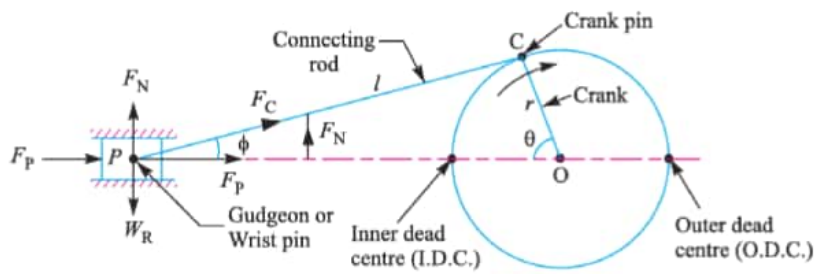

Fig. 16.9. Forces on a connecting rod.

Let
p = Pressure of gas or steam,
A = Area of piston,
m_R = Mass of reciprocating parts,
= Mass of piston, gudgeon pin etc. + $\frac{1}{3}$ rd mass of connecting rod,
ω = Angular speed of crank,
ϕ = Angle of inclination of the connecting rod with the line of stroke,
θ = Angle of inclination of the crank from inner dead centre,
r = Radius of crank,
l = Length of connecting rod, and
n = Ratio of length of connecting rod to radius of crank = l / r.

We know that the force on the piston due to pressure of gas or steam,

$$F_L = \text{Pressure} \times \text{Area} = p \times A$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times {}^*\text{Acceleration} = m_R \times \omega^2 \times r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

It may be noted that in a horizontal engine, reciprocating parts are accelerated from rest during the first half of the stroke (*i.e.* when the piston moves from inner dead centre to outer dead centre). It is then retarted during the latter half of the stroke (*i.e.* when the piston moves from outer dead centre to inner dead centre). The inertia force due to the acceleration of reciprocating parts, opposes the force on the piston. On the other hand, the inertia force due to retardation of the reciprocating parts, helps the force on the piston.

∴ Net force acting on the piston pin (or gudgeon or wrist pin),

$$F_P = \text{Force due to pressure of gas or steam} \pm \text{Inertia force}$$
$$= F_L \pm F_I$$

The –ve sign is used when the piston is accelerated and +ve sign is used when the piston is retarted.

* Acceleration of reciprocating parts = $\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

Scanned with CamScanner

The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls (or normal reaction on crosshead guides). From Fig. 16.9, we see that force in the connecting rod at any instant.

$$F_C = \frac{F_P}{\cos \phi} = \frac{^* F_P}{\sqrt{1 - \frac{\sin^2 \theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (*i.e.* when $\theta = 90°$). But at this position, the gas pressure would be decreased considerably. Thus, for all practical purposes, the force in the connecting rod (F_C) is taken equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects.


Connecting rod.

2. *Inertia bending forces*

Consider a connecting rod *PC* and a crank *OC* rotating with uniform angular velocity ω rad /s. In order to find the acceleration of various points on the connecting rod, draw the Klien's acceleration diagram *CQNO* as shown in Fig. 16.10 (*a*). *CO* represents the acceleration of *C* towards *O* and *NO* represents the acceleration of *P* towards *O*. The acceleration of other points such as *D*, *E*, *F* and *G* etc. on the connecting rod *PC* may be found by drawing horizontal lines from these points to intersect *CN* at *d*, *e*, *f* and *g* respectively. Now *dO*, *eO*, *fO* and *gO* represents the acceleration of *D*, *E*, *F* and *G* all towards *O*. The inertia force acting on each point will be as follows :

Inertia force at $C = m \times \omega^2 \times CO$

Inertia force at $D = m \times \omega^2 \times dO$

Inertia force at $E = m \times \omega^2 \times eO$, and so on.

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to the rod. The parallel (or longitudinal) components adds up algebraically to the force acting on the connecting rod (F_C) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called ***whipping stress.***

* For derivation, please refer to author's popular book on '**Theory of Machines**'.

Scanned with CamScanner

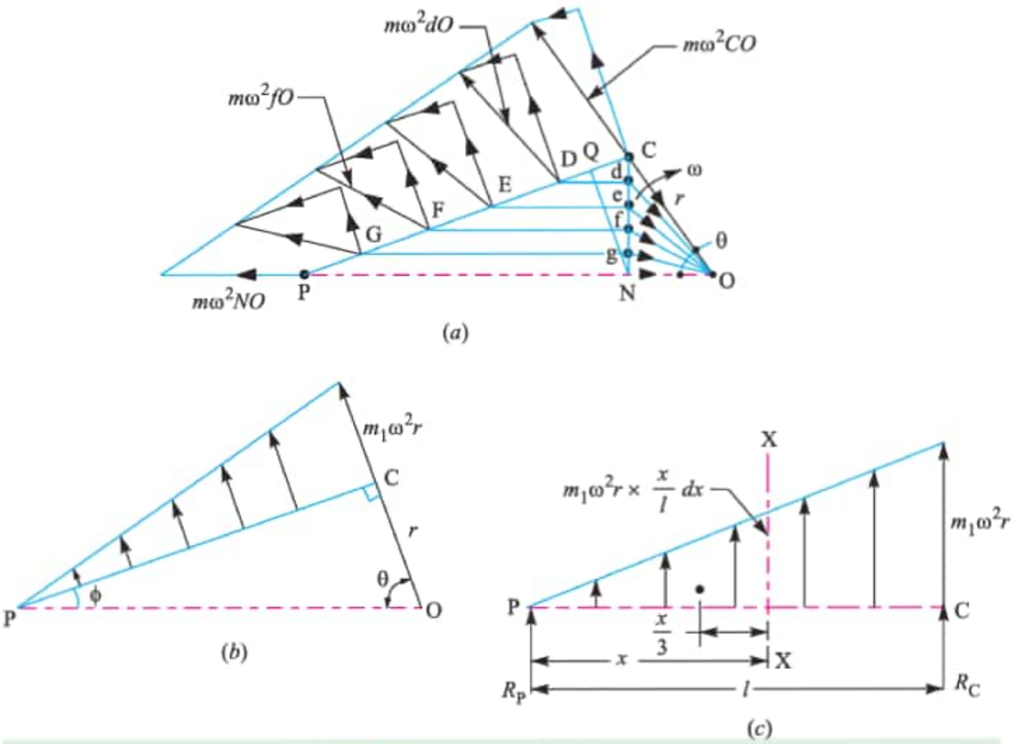

Fig. 16.10. Inertia bending forces.

A little consideration will show that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 16.10 (*b*) and (*c*). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore

Inertia force per unit length at the crank pin

$$= m_1 \times \omega^2 r$$

and inertia force per unit length at the gudgeon pin

$$= 0$$

Inertia forces due to small element of length dx at a distance x from the gudgeon pin P,

$$dF_1 = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

∴ Resultant inertia force,

$$F_1 = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx = \frac{m_1 \times \omega^2 r}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{m_1 \times l}{2} \times \omega^2 r = \frac{m}{2} \times \omega^2 r \qquad \text{... (Substituting } m_1 . l = m\text{)}$$

This resultant inertia force acts at a distance of $2l/3$ from the gudgeon pin P.

Since it has been assumed that $\frac{1}{3}$ rd mass of the connecting rod is concentrated at gudgeon pin P (*i.e.* small end of connecting rod) and $\frac{2}{3}$ rd at the crank pin (*i.e.* big end of connecting rod),

Scanned with CamScanner

therefore the reactions at these two ends will be in the same proportion, *i.e.*

$$R_P = \frac{1}{3} F_I, \text{ and } R_C = \frac{2}{3} F_I$$

Now the bending moment acting on the rod at section *X-X* at a distance *x* from *P*,

$$M_X = R_P \times x - {}^{*}m_1 \times \omega^2 r \times \frac{x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$= \frac{1}{3} F_I \times x - \frac{m_1 . l}{2} \times \omega^2 r \times \frac{x^3}{3 l^2} \qquad \ldots\left(\because R_P = \frac{1}{3} F_I\right)$$

... (Multiplying and dividing the latter expression by *l*)

$$= \frac{F_I \times x}{3} - F_I \times \frac{x^3}{3 l^2} = \frac{F_I}{3}\left(x - \frac{x^3}{l^2}\right) \qquad \ldots(i)$$

For maximum bending moment, differentiate M_X with respect to *x* and equate to zero, *i.e.*

$$\frac{d_{MX}}{dx} = 0 \quad \text{or} \quad \frac{F_I}{3}\left[1 - \frac{3x^2}{l^2}\right] = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \quad \text{or} \quad 3x^2 = l^2 \quad \text{or} \quad x = \frac{l}{\sqrt{3}}$$

Substituting this value of *x* in the above equation (*i*), we have maximum bending moment,

$$M_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}}\right)^2}{l^2}\right] = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}}\right]$$

$$= \frac{F_I}{3} \times \frac{2l}{3\sqrt{3}} = \frac{2 F_I \times l}{9\sqrt{3}}$$

$$= 2 \times \frac{m}{2} \times \omega^2 r \times \frac{l}{9\sqrt{3}} = m \times \omega^2 r \times \frac{l}{9\sqrt{3}} \qquad \ldots\left(\because F_I = \frac{m}{2} \times \omega^2 r\right)$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{max} = \frac{M_{max}}{Z}$$

where Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle θ = 65° to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. In steam engines, even though the pressure is maintained till cut off occurs, the speed is low and therefore the bending stress due to inertia is small. Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (*i.e.* whipping stress).

---

* B.M. due to variable loading from $\left(0 \text{ to } m_1\ \omega^2 r \times \frac{x}{l}\right)$ is equal to the area of triangle multiplied by distance of C.G. from *X-X* $\left(i.e.\ \frac{x}{3}\right)$.

Scanned with CamScanner

Example 16.6. *Determine the dimensions of an I-section connecting rod for a petrol engine from the following data :*

Diameter of the piston	*= 110 mm*
Mass of the reciprocating parts	*= 2 kg*
Length of the connecting rod from centre to centre	*= 325 mm*
Stroke length	*= 150 mm*
R.P.M.	*= 1500 with possible overspeed of 2500*
Compression ratio	*= 4 : 1*
Maximum explosion pressure	*= 2.5 N/mm²*

Connecting rod of a petrol engine.

Solution. Given : $D = 110$ mm $= 0.11$ m ; $m_R = 2$ kg ; $l = 325$ mm $= 0.325$ m ; Stroke length $= 150$ mm $= 0.15$ m ; $N_{min} = 1500$ r.p.m. ; $N_{max} = 2500$ r.p.m. ; *Compression ratio = 4 : 1 ; $p = 2.5$ N/mm²

We know that the radius of crank,

$$r = \frac{\text{Stroke length}}{2} = \frac{150}{2} = 75 \text{ mm} = 0.075 \text{ m}$$

and ratio of the length of connecting rod to the radius of crank,

$$n = \frac{l}{r} = \frac{325}{75} = 4.3$$

We know that the maximum force on the piston due to pressure,

$$F_L = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (110)^2 2.5 = 23\ 760 \text{ N}$$

and maximum angular speed,

$$\omega_{max} = \frac{2\pi \times N_{max}}{60} = \frac{2\pi \times 2500}{60} = 261.8 \text{ rad/s}$$

We know that maximum inertia force of reciprocating parts,

$$F_I = m_R\ (\omega_{max})^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) \qquad \text{... (i)}$$

The inertia force of reciprocating parts is maximum, when the crank is at inner dead centre, *i.e.* when $\theta = 0°$.

$$\therefore \quad F_I = m_R\ (\omega_{max})^2 r \left(1 + \frac{1}{n}\right) \qquad \text{... [From equation (i)]}$$

$$= 2(261.8)^2\ 0.075 \left(1 + \frac{1}{4.3}\right) = 12\ 672 \text{ N}$$

Since the connecting rod is designed by taking the force in the connecting rod (F_C) equal to the maximum force on the piston due to gas pressure (F_L), therefore

Force in the connecting rod,

$$F_C = F_L = 23\ 760 \text{ N}$$

* Superfluous data.

Scanned with CamScanner

Consider the *I*-section of the connecting rod with the proportions as shown in Fig. 16.11. We have discussed in Art. 16.15 that for such a section

$$\frac{I_{xx}}{I_{yy}} = 3.2$$

or $$\frac{k^2_{xx}}{k^2_{yy}} = 3.2, \text{ which is satisfactory.}$$

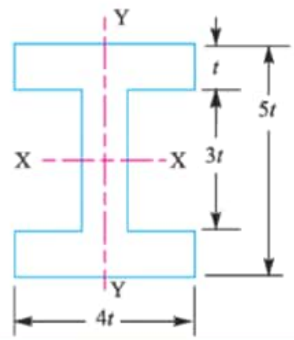

Fig. 16.11

We have also discussed that the connecting rod is designed for buckling about *X*-axis (*i.e.* in a plane of motion of the connecting rod), assuming both ends hinged. Taking a factor of safety as 6, the buckling load,

$$W_{cr} = F_C \times 6 = 23\ 760 \times 6 = 142\ 560 \text{ N}$$

and area of cross-section,

$$A = 2\,(4t \times t) + t \times 3t = 11\,t^2 \text{ mm}^2$$

Moment of inertia about *X*-axis,

$$I_{xx} = \left[\frac{4\,t\,(5\,t)^3}{12} - \frac{3\,t\,(3\,t)^3}{12}\right] = \frac{419\,t^4}{12} \text{ mm}^4$$

∴ Radius of gyration,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419\,t^4}{12} \times \frac{1}{11\,t^2}} = 1.78\,t$$

We know that equivalent length of the rod for both ends hinged,

$$L = l = 325 \text{ mm}$$

Taking for mild steel, σ_c = 320 MPa = 320 N/mm² and a = 1 / 7500, we have from Rankine's formula,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

$$142\ 560 = \frac{320 \times 11\,t^2}{1 + \frac{1}{7500}\left(\frac{325}{1.78\,t}\right)^2}$$

$$40.5 = \frac{t^2}{1 + \frac{4.44}{t^2}} = \frac{t^4}{t^2 + 4.44}$$

or $$t^4 - 40.5\,t^2 - 179.8 = 0$$

∴ $$t^2 = \frac{40.5 \pm \sqrt{(40.5)^2 + 4 \times 179.8}}{2} = \frac{40.5 \pm 48.6}{2} = 44.55$$

... (Taking +ve sign)

or $$t = 6.67 \text{ say } 6.8 \text{ mm}$$

Therefore, dimensions of cross-section of the connecting rod are

Height = $5\,t = 5 \times 6.8 = 34$ mm **Ans.**

Width = $4\,t = 4 \times 6.8 = 27.2$ mm **Ans.**

Scanned with CamScanner

Thickness of flange and web

$$= t = 6.8 \text{ mm} = 0.0068 \text{ m}$$ **Ans.**

Now let us find the bending stress due to inertia force on the connecting rod.

We know that the mass of the connecting rod per metre length,

$$m_1 = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$= A \times l \times \rho = 11\, t^2 \times l \times \rho \qquad ..(\because A = 11\, t^2)$$

$$= 11 (0.0068)^2\, 1 \times 7800 = 3.97 \text{ kg} \qquad ... (\text{Taking } \rho = 7800 \text{ kg/m}^3)$$

∴ Maximum bending moment,

$$M_{max} = m\,\omega^2\, r \times \frac{l}{9\sqrt{3}} = m_1\,\omega^2\, r \times \frac{l^2}{9\sqrt{3}} \qquad ... (\because m = m_1.l)$$

$$= 3.97\, (261.8)^2\, (0.075) \times \frac{(0.325)^2}{9\sqrt{3}} = 138.3 \text{ N-m}$$

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5\,t/2} = \frac{419\,t^4}{12} \times \frac{2}{5\,t} = \frac{419}{30}\, t^3$$

$$= \frac{419}{30}\, (0.0068)^3 = 4.4 \times 10^{-6} \text{ m}^3$$

∴ Maximum bending or whipping stress due to inertia bending forces,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{138.3}{4.4 \times 10^{-6}} = 31.4 \times 10^6 \text{ N/m}^2$$

$$= 31.4 \text{ MPa, which is safe}$$

Note : The maximum compressive stress in the connecting rod will be,

$$\sigma_{c(max)} = \text{Direct compressive stress} + \text{Maximum bending stress}$$

$$= \frac{320}{6} + 31.4 = 84.7 \text{ MPa}$$

Scanned with CamScanner