

ELECTROMAGNETIC WAVES:

The waves are the means of transporting energy (or) information from source to destination. The waves consisting of electric and magnetic fields are called electromagnetic waves.

The electromagnetic waves are said to be in existence if all the four Maxwell's equations are satisfied at the source point [where they are generated], at any point in the medium (through which they travel) and at the destination or load point (where they are received).

Basically the waves radiated from the source are with spherical wavefront, but at large distances from source the spherical waves become practically plane waves.

In general, the wave is a function of time & space.
eg: radio waves, light rays, radar beams, television signals etc.

GENERAL WAVE EQUATION:

To obtain general wave equations, let us assume that the electric and magnetic fields exist in a linear, homogeneous & isotropic medium with the parameters μ , ϵ and σ .

Also assume that the medium is source free which clearly gives the idea about the charge free medium.

Assume that the medium obeys the ohm's law
i.e. $\vec{J} = \sigma \vec{E}$. Then the Maxwell's equation is given by,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{B} = \mu \vec{H} \Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \begin{matrix} \vec{D} = \epsilon \vec{E} \\ \vec{J} = \sigma \vec{E} \end{matrix} \Rightarrow \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \mu \vec{H} \Rightarrow \nabla \cdot \vec{H} = 0 \quad \text{--- (3)}$$

$$\nabla \cdot \vec{E} = 0 \quad \text{--- (4)}$$

Basically the waves radiated from the source are with spherical wavefront, but at large distances from source the spherical waves become practically plane waves.

In general, the wave is a function of time & space.
eg: radio waves, light rays, radar beams, television signals etc.

GENERAL WAVE EQUATION:

To obtain general wave equations, let us assume that the electric and magnetic fields exist in a linear, homogeneous & isotropic medium with the parameters μ , ϵ and σ .

Also assume that the medium is source free which clearly gives the idea about the charge free medium.

Assume that the medium obeys the ohm's law
i.e. $\vec{J} = \sigma \vec{E}$. Then the Maxwell's equation is given by,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{B} = \mu \vec{H} \Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \vec{D} = \epsilon \vec{E} \Rightarrow \nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \mu \vec{H} \Rightarrow \nabla \cdot \vec{H} = 0 \quad \text{--- (3)}$$

$$\nabla \cdot \vec{D} = 0 \Rightarrow \vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = 0 \quad \text{--- (4)}$$

[$\because \rho_v = 0$ for
free space].

To eliminate $\vec{H}$ from (1), taking curl on both sides of equation (1), we get.

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) \quad \text{--- (5)}$$

$\nabla \rightarrow$ indicates differentiation w.r. to space
while $\frac{\partial}{\partial t} \rightarrow$ indicates differentiation w.r. to time.

Both are independent of each other, the operations can be interchanged.

so we get

$$\nabla \times (\nabla \times \vec{E}) = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{H}) \quad \text{--- (6)}$$

sub (2) in (6) we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \left[\sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \right]$$

$$\nabla \times \nabla \times \vec{E} = -\mu \sigma \frac{\partial \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{--- (7)}$$

Now according to vector identity,

$$\nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \quad \text{--- (8)}$$

sub (4) in (8) we get

$$\nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} \quad \text{--- (9)}$$

sub (9) in (7) we get

$$\boxed{+\nabla^2 \vec{E} = +\mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{--- (10)}$$

this is the wave equation for the electric field $\vec{E}$. Now multiplying both the sides of (10) by ϵ , we get

$$\nabla^2 (\epsilon \vec{E}) = \mu \sigma \frac{\partial \epsilon \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \epsilon \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{D} = \mu \sigma \frac{\partial \vec{D}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{D}}{\partial t^2} \quad [\because \vec{D} = \epsilon \vec{E}] \quad \text{--- (11)}$$

This is the wave equation for $\vec{D}$ in uniform medium.

To obtain wave equation of $\vec{H}$, take curl on both sides of (2), we get

$$\nabla \times (\nabla \times \vec{H}) = \nabla \times \sigma \vec{E} + \epsilon \nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{H} = \sigma (\nabla \times \vec{E}) + \epsilon \frac{\partial}{\partial t} (\nabla \times \vec{E}) \quad \text{--- (12)}$$

sub ① in ⑫ we get

$$\begin{aligned}\nabla \times \nabla \times \vec{H} &= \sigma \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) + \epsilon \frac{\partial}{\partial t} \left[-\mu \frac{\partial \vec{H}}{\partial t} \right] \\ &= -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \dots \text{--- (13)}\end{aligned}$$

using vector identity

$$\nabla \times \nabla \times \vec{H} = \nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H} \dots \text{--- (14)}$$

sub ③ in ⑭ we get

$$\nabla \times \nabla \times \vec{H} = -\nabla^2 \vec{H} \dots \text{--- (15)}$$

Equating ⑮ and ⑬ we get

$$-\nabla^2 \vec{H} = -\mu \sigma \frac{\partial \vec{H}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}} \dots \text{--- (16)}$$

This is the wave equation for magnetic field $\vec{H}$. Now multiplying both sides by μ we get

$$\text{⑮} \times \mu$$

$$\nabla^2 (\mu \vec{H}) = \mu \sigma \frac{\partial \mu \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \mu \vec{H}}{\partial t^2}$$

$$\therefore \mu \vec{H} = \vec{B}$$

$$\boxed{\nabla^2 \vec{B} = \mu \sigma \frac{\partial \vec{B}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2}} \dots \text{--- (18)}$$

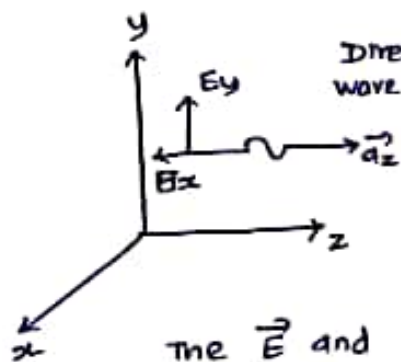
this is the wave equation of $\vec{B}$ in the uniform medium.

UNIFORM PLANE WAVES IN FREE SPACE:

(8)

consider an electromagnetic wave propagating thro' the free space. For free space $\sigma = 0$, consider that the electric field in the wave is in x -direction only while the magnetic field is in the y -direction only.

Both the fields i.e., electric and magnetic field do not vary with x and y but vary only with z . The fields also vary with time as wave propagates in the free space.



Direction of wave propagation.

Basically plane waves means, the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$ lie on the same plane.

The uniform plane wave means the $\vec{E}$ and $\vec{H}$ field vectors are in same plane. Moreover the amplitude and phase of field vectors $\vec{E}$ & $\vec{H}$ is constant over the planes $\parallel^{\text{el}}$ to each other.

Electric field vector is in $\vec{a}_x$ direction, while magnetic field vector is in $\vec{a}_y$ direction. That means $\vec{E}$ & $\vec{H}$ lie in x - y plane. So in any of the planes in the wave, the vectors $\vec{E}$ and $\vec{H}$ are independent of x and y . Thus we can conclude that $\vec{E}$ and $\vec{H}$ are functions of z and t only. Moreover as $\vec{E}$ and $\vec{H}$ are mutually $\perp$ to each other, the electromagnetic waves are also called as transverse electromagnetic waves.

Let us consider wave equations for $\vec{E}$ & $\vec{H}$ fields

Given by,

$$\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \dots \textcircled{1}$$

$$\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2} \quad \dots \textcircled{2}$$

But for free space $\sigma=0$, $N=N_0$ and $\epsilon=\epsilon_0$

Sub these values in (1) & (2) we get

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \text{ --- (3)}$$

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \text{ ---- (4)}$$

Consider eqn (3)

$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \text{ --- (5)}$$

But the wave travels in the z-direction, hence $\vec{E}$ is independent of x and y . Hence first two differential terms in above eqn are zero. Hence we can write

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \text{ ---- (6)}$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \vec{E}}{\partial z^2} \text{ --- (7)}$$

Now according to the results in physics,

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \text{ i.e. } v^2 = \frac{1}{\mu_0 \epsilon_0} = c^2 \text{ where } c = 3 \times 10^8 \text{ m/s} = \text{velocity of light}$$

Sub above relations in (7) we get

$$\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = v^2 \frac{\partial^2 \vec{E}}{\partial z^2}} \text{ --- (8)}$$

Above equation is other form of wave equation.

Similarly to this we can write.

$$\boxed{\frac{\partial^2 \vec{H}}{\partial t^2} = v^2 \frac{\partial^2 \vec{H}}{\partial z^2}} \text{ ---- (9)}$$

Reconsidering eqn (6)

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

For the wave propagating in z-direction, $\vec{E}$ may have E_x & E_y ⑨
 Component definitely not E_z . According to assumption, $\vec{E}$ is in $\vec{a}_x$
 direction, so let us consider that only E_x is present. Then we
 can rewrite above equation as,

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \text{ ---- ⑩}$$

Let $E_x = E_m e^{j\omega t}$

where $E_m \rightarrow$ Amplitude of the electric field
 $\omega \rightarrow$ Angular freq.

Partially differentiating E_x twice w.r. to t , we get

$$\frac{\partial^2 E_x}{\partial t^2} = E_m (j\omega)(j\omega) e^{j\omega t} = -\omega^2 E_m e^{j\omega t}$$

But $E_m e^{j\omega t} = E_x$

$$\therefore \frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_x$$

Sub above eqn in ⑩ we get

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 [-\omega^2 E_x] = -\omega^2 \mu_0 \epsilon_0 E_x \text{ ---- ⑪}$$

Let $\frac{\partial}{\partial z} = D$ i.e. $\frac{\partial^2}{\partial z^2} = D^2 \therefore$ ⑪ becomes.

$$D^2 E_x + \omega^2 \mu_0 \epsilon_0 E_x = 0$$

Thus Auxiliary Equation becomes,

$$(D^2 + \omega^2 \mu_0 \epsilon_0) E_x = 0$$

Hence equating bracket terms to zero, we get

$$D^2 + \omega^2 \mu_0 \epsilon_0 = 0$$

$$(or) D^2 = -\omega^2 \mu_0 \epsilon_0$$

$$(or) D = \pm j\omega \sqrt{\mu_0 \epsilon_0} = \pm j\beta$$

where $\beta = \omega \sqrt{\mu_0 \epsilon_0}$ which is called Phase shift constant
 measured in rad/m.

Hence the solution of eqn (11) can be written as,

$$E_x = K_1 e^{-j\omega\sqrt{\mu_0\epsilon_0}z} + K_2 e^{j\omega\sqrt{\mu_0\epsilon_0}z}$$

$$E_x = K_1 e^{-j\beta z} + K_2 e^{j\beta z} \quad \text{--- (12)}$$

Let K_1 and K_2 be the constants w.r.to z but are functions of t . Let us assume K_1 and K_2 as

$$K_1 = E_m e^{+j\omega t} \quad \&$$

$$K_2 = E_m e^{-j\omega t}$$

sub K_1 and K_2 in (12) we get

$$\begin{aligned} E_x &= E_m e^{+j\omega t} e^{-j\beta z} + E_m e^{-j\omega t} e^{j\beta z} \\ &= E_m e^{j(\omega t - \beta z)} + E_m e^{-j(\omega t + \beta z)} \quad \text{--- (13)} \end{aligned}$$

To find the electric field in the time domain, taking real part of (13), we get

$$E_x = \text{Re} \left[E_m e^{+j(\omega t - \beta z)} + E_m e^{-j(\omega t + \beta z)} \right]$$

travelling in +z direction $\leftarrow$ $E_x = E_m \cos(\omega t - \beta z) + E_m \cos(\omega t + \beta z)$ V/m. $\rightarrow$ travelling in -ve z-direction.

Above equation is the sinusoidal function consisting of two components. one in forward direction and other in backward direction

POYNTING VECTOR & POYNTING THEOREM:

By means of Electromagnetic (EM) waves, an energy can be transported from transmitter to receiver. The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of Poynting theorem.

As we know $\vec{E}$ & $\vec{H}$ are basic fields, $\vec{E}$ is electric field expressed in V/m ; while $\vec{H}$ is magnetic field measured in A/m . So if we take dot product of the two fields, dimensionally we get a unit $V \cdot A / m^2$ (or) $\frac{Watt}{m^2}$. So this product of $\vec{E}$ & $\vec{H}$ gives a new quantity which is expressed as $Watt/m^2$. This quantity is called Power density.

As $\vec{E}$ & $\vec{H}$ are vectors, to get power density we may carry out either dot product or cross product. The result of dot product is always a scalar quantity. But as power flows in certain directions, it is a vector quantity. To illustrate this, consider that the field is transmitted in the form of an electromagnetic waves from an antenna. Both the fields are sinusoidal in nature.

The power radiated from antenna has a particular direction. Hence to calculate a power density, we must carry out a cross product of $\vec{E}$ and $\vec{H}$.

The power density is given by

$$\vec{P} = \vec{E} \times \vec{H}$$

where $\vec{P}$ is called Poynting vector.

Poynting Theorem is based on the law of conservation of energy in electromagnetism. Poynting Theorem can be stated as,

The net power flowing out of a ^{given} volume V is equal to the time rate of decrease in the energy stored within volume V minus the ohmic power dissipated.

Suppose

$$\vec{E} = E_x \vec{a}_x$$

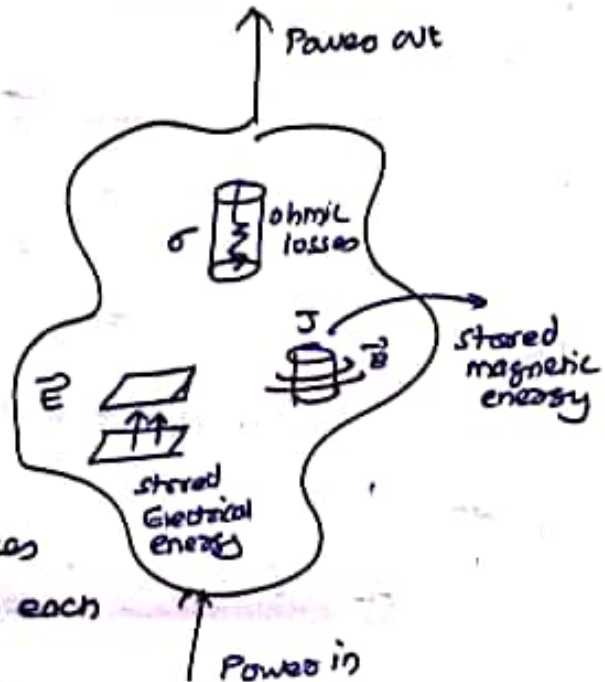
$$\vec{H} = H_y \vec{a}_y \text{ then}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= E_x \vec{a}_x \times H_y \vec{a}_y$$

$$\boxed{\vec{P} = E_x H_y \vec{a}_z}$$

The above evaluation indicates $\vec{E}$, $\vec{H}$ and $\vec{P}$ are mutually $\perp$ to each other.



Consider that the electric field propagates in free space given by

$$\vec{E} = [E_m \cos(\omega t - \beta z)] \vec{a}_x$$

In the medium, the ratio of magnitudes of $\vec{E}$ & $\vec{H}$ depends on its intrinsic impedance η , for free space.

$$\eta = \eta_0 = \frac{E_m}{H_m} = 120\pi = 377 \Omega.$$

Moreover in free space, electromagnetic wave travels at a speed of light,

thus we can write,

$$\vec{H} = [H_m \cos(\omega t - \beta z)] \vec{a}_y$$

$$= \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z) \right] \vec{a}_y$$

According to Poynting theorem.

(11)

$$\vec{P} = \vec{H} \times \vec{E}$$

$$= [E_m \cos(\omega t - \beta z)] \vec{a}_x \times \left[\frac{E_m}{\eta_0} \cos(\omega t - \beta z) \right] \vec{a}_y$$

$$\vec{P} = \frac{E_m^2}{\eta_0} \cos^2(\omega t - \beta z) \vec{a}_z \text{ W/m}^2$$

This is nothing but the power density measured in Watt/m^2 . Thus the power passing particular area is given by,

$$\text{Power} = \text{Power density} \times \text{Area}.$$

AVERAGE POWER DENSITY (P_{avg}).

To find average power density, let us integrate by power density in z-direction over one cycle and divide the period T of one cycle.

$$\therefore P_{avg} = \frac{1}{T} \int_0^T \frac{E_m^2}{\eta} \cos^2(\omega t - \beta z) dt.$$

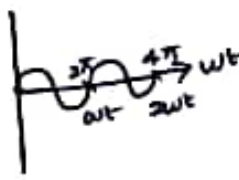
$$= \frac{E_m^2}{\eta T} \int_0^T \cos^2(\omega t - \beta z) dt$$

$$= \frac{E_m^2}{\eta T} \int_0^T \frac{1 + \cos 2(\omega t - \beta z)}{2} dt$$

$$= \frac{E_m^2}{\eta T} \left[\frac{t}{2} + \frac{\sin 2(\omega t - \beta z)}{(2\omega) 2} \right]_0^T$$

$$= \frac{E_m^2}{\eta T} \left[\frac{t}{2} + \frac{\sin(2\omega t - 2\beta z)}{4\omega} \right]_0^T$$

$$= \frac{E_m^2}{\eta T} \left[\frac{T}{2} + \frac{\sin(2\omega T - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega} \right]$$



$$= \frac{E_m^2}{\eta T} \left[\frac{T}{2} + \frac{\sin(4\pi - 2\beta z)}{4\omega} + \frac{\sin(2\beta z)}{4\omega} \right]$$

$$= \frac{E_m^2}{\eta T} \left[\frac{T}{2} - \frac{\sin 2\beta z}{4\omega} + \frac{\sin 2\beta z}{4\omega} \right] = \frac{E_m^2}{2\eta} = \frac{E_m^2}{2\eta}$$

Hence the average power is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \text{ W/m}^2$$

INTEGRAL & POINT FORMS OF POYNTING THEOREM:

Consider Maxwell's equations as given below,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

Dotting both the sides of eqn (2) with $\vec{E}$, we get

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E} \cdot (\sigma \vec{E}) + \vec{E} \cdot \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right) \quad \text{--- (3)}$$

Let us make use of vector identity as given below

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

Applying the above identity to eqn (3) with $\vec{A} = \vec{E}$ & $\vec{B} = \vec{H}$ we get

PROVE:

$$\vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{E} \times \vec{H})$$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot (\nabla \times \vec{H})$$

Sub (3) in above equation we get

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot (\sigma \vec{E}) + \vec{E} \cdot \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \vec{E} \cdot \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right) \quad \text{--- (4)}$$

Consider the first term of (4) we get, & by substituting (1) we get

$$\vec{H} \cdot (\nabla \times \vec{E}) = \vec{H} \cdot \left(-\mu \frac{\partial \vec{H}}{\partial t} \right)$$

$$\vec{H} \cdot (\nabla \times \vec{E}) = -\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \text{--- (i)}$$

Now consider eqn (i),

$$\frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) = \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

$$\frac{\partial}{\partial t} H^2 = 2\vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

$$\frac{1}{2} \frac{\partial}{\partial t} H^2 = \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \dots (ii)$$

Similarly we can write

$$\frac{1}{2} \frac{\partial}{\partial t} E^2 = \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad \dots (iii)$$

Substituting (i), (ii) & (iii) in (4) we get

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \vec{E} \cdot \left(\epsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$- \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \epsilon \left[\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right]$$

$$- \frac{\mu}{2} \frac{\partial}{\partial t} H^2 - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \frac{\epsilon}{2} \frac{\partial}{\partial t} E^2$$

$$- \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \frac{\epsilon}{2} \frac{\partial}{\partial t} E^2 + \frac{\mu}{2} \frac{\partial}{\partial t} H^2$$

$$- \nabla \cdot (\vec{E} \times \vec{H}) = \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} [\mu H^2 + \epsilon E^2]$$

But $\vec{E} \times \vec{H} = \vec{P}$

$$\therefore - \nabla \cdot (\vec{P}) = \sigma E^2 + \frac{1}{2} \frac{\partial}{\partial t} [\mu H^2 + \epsilon E^2] \quad \dots (5)$$

The above equation represents Poynting Theorem in Point form.

If we integrate this power over a volume, we can get energy distribution as,

$$- \int_V \nabla \cdot \vec{P} \, dV = \int_V \sigma E^2 \, dV + \frac{\partial}{\partial t} \int_V \frac{1}{2} [\mu H^2 + \epsilon E^2] \, dV$$

Applying divergence theorem to left of above equation we get

$$- \oint \vec{P} \cdot d\vec{s} = \int_V \sigma E^2 \, dV + \frac{\partial}{\partial t} \int_V \frac{1}{2} [\mu H^2 + \epsilon E^2] \, dV$$

Above equation represents Poynting theorem in integral form.

REFLECTION OF UNIFORM PLANE WAVES:

We have so far, studied the uniform plane waves travelling in unbounded and homogeneous media. But practically, very often, wave propagates in boundary regions consisting several media of different constitutive parameters such as $\epsilon, \mu, \sigma, \eta$ etc.

Before we actually start with the reflection of the uniform plane wave, let us consider simple example of a transmission line.

Consider a transmission line having a characteristic impedance Z_0 . Assume that the line is terminated in load impedance Z_L .

If the load impedance Z_L equals the characteristic impedance Z_0 (i.e. $Z_L = Z_0$), then the line is said to be properly terminated.

If $Z_L \neq Z_0$, then there is a mismatch b/w the two impedances and the line is not properly terminated. Consider that the wave travelling along the line incidents at the load. The part of the wave gets absorbed by the load, while the other part is reflected back to the generator.

So we can say reflection occurs at the load if $Z_L \neq Z_0$. If there are two waves, one incident in forward direction, while other reflected back in backward direction, then the standing waves are said to be produced along the line.

When a uniform plane wave travels from one medium to other having different intrinsic impedances, The reflection takes place at the boundaries.

The part of the wave is transmitted in medium 2 and remaining part is reflected back to medium 1, depending upon the consecutive parameters of media.

Depending upon the manner in which the uniform plane wave is incident on the boundary, there are two cases of incidence.

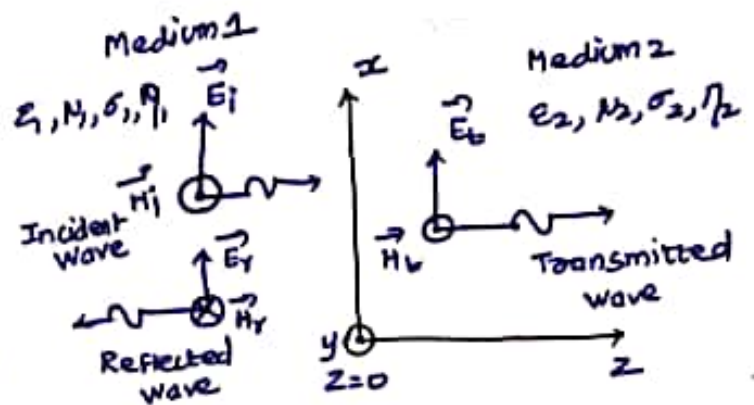
(i) Normal incidence

(ii) oblique incidence.

NORMAL INCIDENCE AT PLANE DIELECTRIC BOUNDARY:

Consider a uniform plane wave striking the interface b/w the two dielectrics at right angles as shown in the figure.

Assume that the uniform plane wave travels along $+z$ direction and incidence at right angles at the boundary b/w two dielectric media i.e. at $z=0$.



Below $z=0$, let the properties of medium 1 be $\epsilon_1, \mu_1, \sigma_1, \eta_1$ and above $z=0$, the properties of medium 2 be $\epsilon_2, \mu_2, \sigma_2, \eta_2$

So depending upon the properties of two media, part of the wave will be transmitted in medium 2 while other part will be reflected back in medium 1

Let E_i & H_i be the field strengths of the incident wave striking at the boundary.

E_t & H_t be the field strengths of the transmitted wave in the medium 2.

E_r & H_r be the field strengths of the reflected wave in the medium 1 returning back from the interface.

from figure it is clear that in medium 1, the total field comprises of both the incident and reflected fields. But in medium 2 only transmitted field gives the total field.

So the conditions for the total field in medium 1 are given by,

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r \quad \&$$

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r$$

iii) The conditions for total field in medium 2 is given by,

$$\vec{E}_2 = \vec{E}_t \quad \&$$

$$\vec{H}_2 = \vec{H}_t$$

According to the boundary condition, the tangential components of $\vec{E}$ & $\vec{H}$ must be continuous at the interface $z=0$.

$$\therefore \vec{E}_{1tan} = \vec{E}_{2tan}$$

$$\vec{H}_{1tan} = \vec{H}_{2tan}$$

Thus at interface $z=0$, we can write

$$\vec{E}_i + \vec{E}_r = \vec{E}_t \quad \&$$

$$\vec{H}_i + \vec{H}_r = \vec{H}_t$$

The relationships b/w the magnitude of $\vec{E}$ & $\vec{H}$ at $z=0$ are given by the following expressions

$$E_i = \eta_1 H_i$$

$E_r = -\eta_1 H_r$ as direction of reflected wave is opposite to that of incident wave

$$E_t = \eta_2 H_t$$

In terms of magnitudes of the fields $\vec{E}$ & $\vec{H}$ at the interface, we can write

$$E_i + E_r = E_t \quad \text{--- (1)}$$

$$H_i + H_r = H_t \quad \text{--- (2)}$$

In eqn (2), putting the values of H_i , H_r and H_t in terms of E_i , E_r & E_t we get

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$\therefore E_i - E_r = \frac{\eta_1}{\eta_2} E_t \quad \text{--- (3)}$$

Adding eqn (1) & (3), we get

$$2E_i = \left(1 + \frac{\eta_1}{\eta_2}\right) E_t$$

$$2E_i = \left(\frac{\eta_1 + \eta_2}{\eta_2}\right) E_t$$

$$E_t = \frac{2\eta_2}{\eta_1 + \eta_2} E_i \quad \text{--- (4)}$$

The Transmission coefficient is denoted by τ and it is given by,

$$\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \text{--- (5)}$$

Eliminating E_t from eqn (1) & (3), we get

$$\frac{(1)}{(3)} \Rightarrow \frac{E_i + E_r}{E_i - E_r} = \frac{\eta_2}{\eta_1} \Rightarrow E_i + E_r = \frac{\eta_2}{\eta_1} E_i - E_r$$

$$\eta_2(E_i + E_r) = \eta_1(E_i - E_r)$$

~~$$\eta_1 E_i + \eta_2 E_r = \eta_2 E_i - \eta_1 E_r$$~~

$$\eta_1 E_i + \eta_2 E_r = \eta_2 E_i - \eta_1 E_r$$

~~$$E_i(\eta_1 - \eta_2) = -E_r(\eta_1 + \eta_2)$$~~

$$(\eta_1 - \eta_2) E_i = -E_r (\eta_1 + \eta_2)$$

~~$$E_i$$~~

$$E_i (\eta_1 - \eta_2) = -E_r (\eta_1 + \eta_2)$$

$$E_r = \frac{\eta_1 - \eta_2}{-(\eta_1 + \eta_2)} E_i$$

$$E_r = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} E_i \quad \text{--- (6)}$$

The reflection co-efficient is denoted as Γ and it is given by,

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} \quad \text{--- (7)}$$

from eqn (5) and (7), we can draw some important results as

(a) $1 + \Gamma = \tau$

(b) $0 \leq |\Gamma| \leq 1$

(c) Both the co-efficients Γ and τ are dimensionless and may be complex in nature.

According to Poynting Theorem, The average Power density is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \text{ W/m}^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average Power ~~transmitted~~ ^{incident} in medium-1 is given by

$$P_{iavg} = \frac{1}{2} \frac{E_i^2}{\eta_1} \text{ W/m}^2$$

The average Power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2} \frac{E_r^2}{\eta_1} \text{ W/m}^2$$

The ratio of Power transmitted to Power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{\frac{1}{2} E_t^2 / \eta_2}{\frac{1}{2} E_i^2 / \eta_1} = \frac{\eta_1}{\eta_2} \left[\frac{E_t}{E_i} \right]^2 = \frac{\eta_1}{\eta_2} \left[\frac{2\eta_2}{\eta_2 + \eta_1} \right]^2 = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2} P_{iavg} \dots \textcircled{1}$$

$$= \frac{\eta_1^2 + 2\eta_1\eta_2 + \eta_2^2 - (\eta_2^2 + 2\eta_1\eta_2 + \eta_1^2)}{(\eta_1 + \eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1 + \eta_2)^2 - (\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} P_{iavg}$$

$$= \left[\frac{(\eta_1 + \eta_2)^2}{(\eta_1 + \eta_2)^2} - \frac{(\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} \right] P_{iavg}$$

$$P_{tavg} = [1 - |\Gamma|^2] P_{iavg} \dots \textcircled{2}$$

The ratio of Power reflected to Power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{\frac{1}{2} E_r^2 / \eta_1}{\frac{1}{2} E_i^2 / \eta_1} = \left[\frac{E_r}{E_i} \right]^2 = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right]^2 = \frac{(\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} \dots \textcircled{3}$$

Rearranging terms we get

$$P_{\text{avg}} = \frac{(\eta_2 - \eta_1)^2}{(\eta_2 + \eta_1)^2} P_{\text{avg}}$$

$$\therefore \boxed{P_{\text{avg}} = (\Gamma)^2 P_{\text{avg}}} \quad \text{--- (4)}$$

Adding (1) and (3) we get

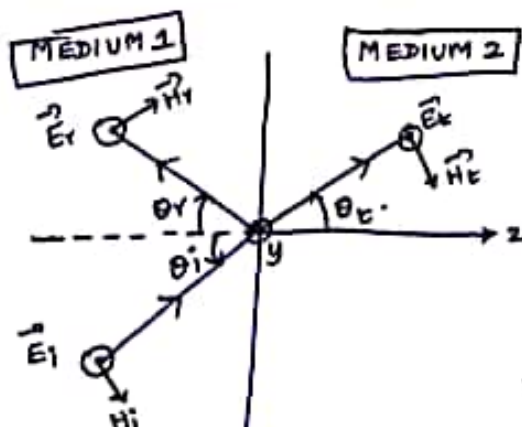
$$\begin{aligned} \frac{P_{\text{avg}}}{P_{\text{avg}}} + \frac{P_{\text{avg}}}{P_{\text{avg}}} &= \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2} + \frac{(\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} \\ &= \frac{\eta_1^2 + \eta_2^2 + 2\eta_1\eta_2}{(\eta_1 + \eta_2)^2} = \frac{(\eta_1 + \eta_2)^2}{(\eta_1 + \eta_2)^2} \\ &= 1 \end{aligned}$$

$$\therefore \boxed{P_{\text{avg}} + P_{\text{avg}} = P_{\text{avg}}} \quad \text{--- (5)}$$

OBLIQUE INCIDENCE:

When a uniform plane wave strikes obliquely on the surface (either conductor or dielectric), the behaviour of the reflected wave is decided by the polarization of the incident wave. There are two cases for the oblique incidence as given below.

case (i): The electric field vector $\perp$ to the plane of incidence. In other words, the Electric field vector is aligned $\parallel$ to the boundary surface as shown below. This is called Horizontal Polarization.



According to Poynting Theorem, The average Power density is given by,

$$P_{avg} = \frac{1}{2} \frac{E_m^2}{\eta} \text{ W/m}^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average Power ~~transmitted~~ ^{incident} in medium-1 is given by

$$P_{iavg} = \frac{1}{2} \frac{E_i^2}{\eta_1} \text{ W/m}^2$$

The average Power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2} \frac{E_r^2}{\eta_1} \text{ W/m}^2$$

The ratio of Power transmitted to Power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{\frac{1}{2} E_t^2 / \eta_2}{\frac{1}{2} E_i^2 / \eta_1} = \frac{\eta_1}{\eta_2} \left[\frac{E_t}{E_i} \right]^2 = \frac{\eta_1}{\eta_2} \left[\frac{2\eta_2}{\eta_2 + \eta_1} \right]^2 = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1 + \eta_2)^2} P_{iavg} \dots \textcircled{1}$$

$$= \frac{\eta_1^2 + 2\eta_1\eta_2 + \eta_2^2 - (\eta_2^2 - 2\eta_1\eta_2 + \eta_1^2)}{(\eta_1 + \eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1 + \eta_2)^2 - (\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} P_{iavg}$$

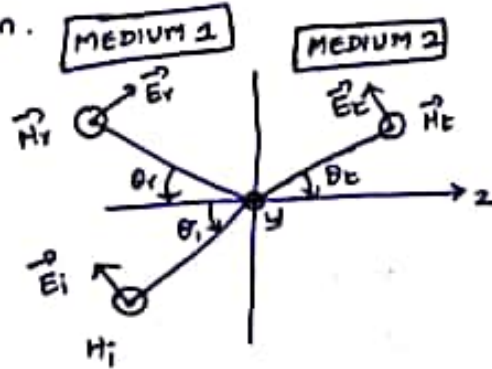
$$= \left[\frac{(\eta_1 + \eta_2)^2}{(\eta_1 + \eta_2)^2} - \frac{(\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} \right] P_{iavg}$$

$$P_{tavg} = [1 - |\Gamma|^2] P_{iavg} \dots \textcircled{2}$$

The ratio of Power reflected to Power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{\frac{1}{2} E_r^2 / \eta_1}{\frac{1}{2} E_i^2 / \eta_1} = \left[\frac{E_r}{E_i} \right]^2 = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \right]^2 = \frac{(\eta_2 - \eta_1)^2}{(\eta_1 + \eta_2)^2} \dots \textcircled{3}$$

CASE (ii) : The magnetic field vector is aligned $\parallel$ to the boundary surface. In other words, The magnetic field vector is $\perp$ to the plane of incidence while electric field vector is aligned $\parallel$ to the plane of incidence as shown below. This is called vertical Polarization.



PLANE OF INCIDENCE:-

A plane of incidence is a plane containing the vector in the direction of propagation of the incident wave and the normal to the boundary surface.

POLARIZATION OF ELECTROMAGNETIC WAVES:

The Polarization of uniform plane waves is defined as time varying behaviour of the Electric field intensity vector $\vec{E}$ at some fixed point in space, along the direction of propagation.

There are three different types of Polarization of a uniform plane wave as given below

- (a) Linear Polarization
- (b) Elliptical Polarization
- (c) Circular Polarization.

In other words Polarization is nothing but the way in which the magnitude and direction of the electric field varies.

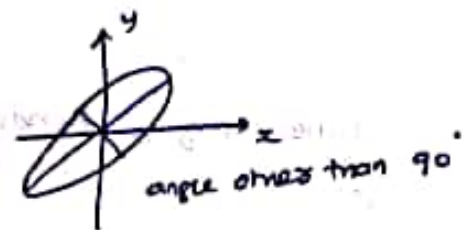
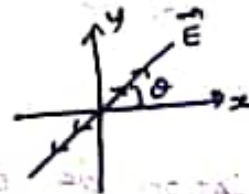
LINEAR POLARIZATION:

Let the components of $\vec{E}$ be $\vec{E}_x$ and $\vec{E}_y$ along x and y -direction respectively. Both these components are in phase having different amplitudes. As $\vec{E}_x$ and $\vec{E}_y$ are in phase they will have their amplitudes reaching max or min value simultaneously. Also if the amplitude of $\vec{E}_x$ increases or decreases the amplitude of $\vec{E}_y$ also increases or decreases.

In other words, at any point along the z -axis the ratio of amplitudes of both of the components is constant as both of them are in phase having same wavelength.

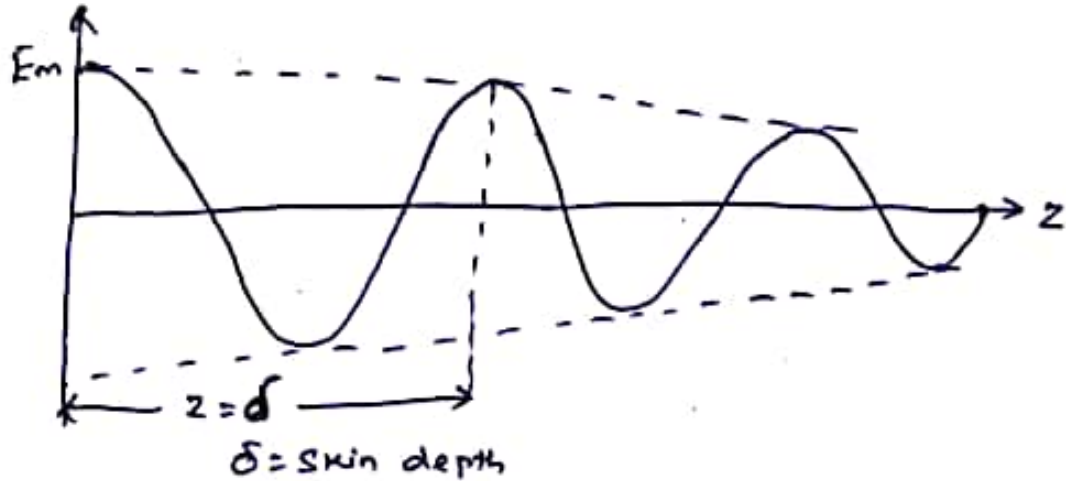
The electric field $\vec{E}$ is the resultant of $\vec{E}_x$ & $\vec{E}_y$ and the direction of it depends on the relative magnitude of $\vec{E}_x$ & $\vec{E}_y$. Thus the angle made by $\vec{E}$ with x -axis is given by,

$$\theta = \tan^{-1} \frac{E_y}{E_x}$$



SKIN DEPTH:

The skin depth is defined as the depth in which the wave has attenuated to $1/e$ i.e., approximately 37% of its original value. It is also called as depth of penetration.



Problem:

① A 300 Hz uniform plane wave propagate through fresh water for which $\sigma = 0$, $\mu_r = 1$, $\epsilon_r = 78$. Calculate wavelength.

Given $f = 300 \text{ Hz}$, $\sigma = 0$, $\mu_r = 1$, $\epsilon_r = 78$

$$\beta = \omega \sqrt{\mu \epsilon} = \omega \sqrt{(\mu_0 \mu_r)(\epsilon_0 \epsilon_r)}$$

$$\omega = 2\pi f$$

$$\beta = (2 \times \pi \times 300) \times (4\pi \times 10^{-7})^{1/2} (8.854 \times 10^{-12})^{1/2} (78)$$

$$\beta = 5.54 \times 10^{-4} \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}$$

$$\beta = 5.54 \times 10^{-4} \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}$$

$$\lambda = 1.13 \times 10^{-4} \text{ m or } \mu\text{m}$$