MAXWELL'S EQUATION:

we have seen that a static electric field $\vec{E}$ can exist without a magnetic field $\vec{H}$ demonstrated by a capacitor with a static charge Q.

IIIly a conductor with a constant current I has a magnetic field $\vec{H}$ in the absence of an electric field $\vec{E}$.

But in the case of time varying fields, $\vec{E}$ & $\vec{H}$ doesnot exists without each other.

Maxwell's Equations are nothing but a set of four expressions derived from Ampere's circuital law, Faraday's law, Gauss's law for electric field and Gauss's law for magnetic field.

MAXWELL'S EQUATION FOR STATIC FIELDS:

MAXWELL'S EQUATION DERIVED FROM FARADAY'S LAW:

A] MAXWELL'S EQUATION DERIVED

According to the basic concept from electrostatic field, the workdone over a closed path (or) closed contour (i.e. starting point same as terminating point) is always zero.

Mathematically it is represented as,

$$\oint \vec{E} \cdot \vec{dL} = 0.$$

The above equation is called integral form of maxwell's Equation derived from Faraday's law of static field.

Now using stoke's theorem converting the close line integral into the surface integral we get,

$$\oint \vec{E} \cdot \vec{dL} = \int_S (\nabla \times \vec{E}) \cdot \vec{ds} = 0$$

$$\therefore \int_S (\nabla \times \vec{E}) \cdot \vec{ds} = 0$$

But $\vec{ds}$ cannot be zero (i.e $\vec{ds} \neq 0$) That means,

$$\boxed{\nabla \times \vec{E} = 0} \rightarrow$$ Point form of Maxwell's eqn derived from Faraday's law of static fields.
or
differential form

MAXWELL'S EQUATION DERIVED FROM AMPERE'S CIRCUITAL LAW:

According to basic concept of magnetostatics an Ampere's circuital law states that the line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path. Mathematically it is given as,

$$\oint \vec{H} \cdot \vec{dL} = I$$

Now the current enclosed is equal to the product of current density normal to the closed path and area of closed path. Hence we get,

$$I = \int_S \vec{J} \cdot \vec{ds} \quad \text{where} \quad \vec{J} = \text{current density.}$$

Hence equating above equations we get,

$$\oint \vec{H} \cdot \vec{dL} = \int_S \vec{J} \cdot \vec{ds}$$

This above expression is called Integral form of maxwell's equation from Ampere circuital law for static field.

Now by applying stoke's theorem, L.H.S of above equation can be converted into surface Integral

$$\therefore \oint \vec{H} \cdot \vec{dL} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds}$$

Hence we get

$$\boxed{\nabla \times \vec{H} = \vec{J}}$$

differential form of Maxwell's eqn derived from Ampere circuital law for static field.

Maxwell's Equation derived from Gauss's law for Electrostatic fields.

According to Gauss's law of electrostatic fields, the electric flux passing thro' any closed surface is equal to the Total charge enclosed by that surface. Mathematically we can write,

$$\psi = \oint \vec{D} \cdot \vec{ds} = Q_{enclosed} \quad \text{---} (1)$$

The most common form to represent Gauss's law mathematically is with volume charge density ρ_v. Hence we can write,

$$\oint \vec{D} \cdot \vec{ds} = \int_v \rho_v \, dv \quad \text{---} (2)$$

The above equation is called integral form of maxwell's equation derived from Gauss's law for static electric field.

To establish relationship b/w $\vec{D}$ and ρ_v, converting closed surface integral into volume integral using divergence Theorem as,

$$\oint \vec{D} \cdot \vec{ds} = \int_v (\nabla \cdot \vec{D}) \cdot dv \quad \text{---} (3)$$

comparing (2) and (3) we get

$$\int_v (\nabla \cdot \vec{D}) \cdot dv = \int_v \rho_v \, dv$$

$\rightarrow \nabla \cdot \vec{D} = \rho_v \longrightarrow$ Point form or differential form of Maxwell's Equation derived from Gauss's law for static electric field.

MAXWELL'S EQUATION DERIVED FROM GAUSS'S LAW FOR MAGNETOSTATIC FIELD:

According to Gauss's law for magnetostatic field, the magnetic flux cannot reside in a closed surface due to non existance of single magnetic Pole.

Mathematically we can write,

$$\oint_s \vec{B} \cdot \vec{ds} = 0.$$

The above equation is called Integral form of Maxwell's Equation derived from Gauss's law for static magnetic field.

Now using divergence Theorem, we can write

$$\oint_S \vec{B}.\vec{ds} = \int_V (\nabla.\vec{B})\, dv = 0$$

(ie) $\int_V (\nabla.\vec{B})\, dv = 0$

Now dv cannot be zero that means

$\nabla.\vec{B} = 0$

↳ Point form or differential form of maxwell's equation derived from Gauss's law for static magnetic field.

MAXWELL'S EQUATION FOR TIME VARYING FIELD:

MAXWELL'S EQUATION DERIVED FROM FARADAY'S LAW:

Now consider Faraday's law which relates e.m.f induced in a circuit to a circuit to the time rate of change of decrease of total magnetic flux linking the ckt. Thus we can write.

$$\oint \vec{E}.\vec{dL} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{ds} \quad \text{--- (1)}$$

→ Maxwell's Eqn derived from Faraday's law expressed in Integral form.

The Total electromotive force (e.m.f) induced in a closed path is equal to the negative surface integral of the rate of change of flux density w.r.to time over an entire surface bounded by the same closed path.

using Stoke's theorem, we get

$$\int_S (\nabla\times\vec{E}).\vec{ds} = -\int_S \frac{\partial \vec{B}}{\partial t}.\vec{ds}$$

$$\therefore \quad \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

→ Point form or differential form derived from Faraday's law.

MAXWELL'S EQUATION DERIVED FROM AMPERE'S CIRCUITAL LAW:

According to Ampere's circuital law, The line integral of magnetic field Intensity $\vec{H}$ around a closed path is equal to current enclosed by the path.

$$\oint \vec{H} \cdot \vec{dL} = I_{enclosed}.$$

$$I_{enclosed} = \int_S \vec{J} \cdot \vec{ds}$$

$$\therefore \oint \vec{H} \cdot \vec{dL} = \int_S \vec{J} \cdot \vec{ds}$$

Above expression can be made further general by adding displacement current density to conduction current density as follows,

$$\oint \vec{H} \cdot \vec{dL} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t}\right] \cdot \vec{ds} \rightarrow \text{Integral form}$$

Applying stokes theorem.

$$\oint \vec{H} \cdot \vec{dL} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \left[\vec{J_c} + \frac{\partial \vec{D}}{\partial t}\right] \cdot \vec{ds}$$

$$\therefore (\nabla \times \vec{H}) = \vec{J_c} + \frac{\partial \vec{D}}{\partial t} \rightarrow \text{Point form.}$$

Statement:

The Total MMF around any closed Path is equal to the surface integral of the conduction and displacement current densities over the entire surface bounded by the same closed path.

Total flux leaving out of a closed surface is equal to the total charge enclosed by a finite volume.

The surface integral of magnetic flux density over a closed surface is always equal to zero.

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Differential form	Integral form	Significance
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$	Faraday's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \vec{dL} = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot ds$	Ampere circuital law
$\nabla \cdot \vec{D} = \rho_v$	$\oint \vec{D} \cdot \vec{ds} = \int_s \rho_v \, dV$	Gauss's law
$\nabla \cdot \vec{B} = 0$	$\oint_s \vec{B} \cdot \vec{ds} = 0$	No isolated magnetic charges.

MAXWELL'S EQUATION FOR FREE SPACE:

Free space is a non-conducting medium in which volume charge density $\rho_v = 0$ and conductivity $\sigma = 0$

Point form	Integral form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$
$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ $[\because \vec{J} = \sigma \vec{E}]$	$\oint \vec{H} \cdot \vec{dL} = \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$
$\nabla \cdot \vec{D} = 0$	$\oint \vec{D} \cdot \vec{ds} = 0$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$.

MAXWELL'S EQUATION for GOOD conductors:

for good conductors $\vec{J} >> \frac{\partial \vec{D}}{\partial t}$ & $\rho_v = 0$.

$\therefore$ $\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \vec{dL} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds}$
$\nabla \times \vec{H} = \vec{J}$	$\oint \vec{H} \cdot \vec{dL} = \int_s \vec{J} \cdot \vec{ds}$
$\nabla \cdot \vec{D} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$
$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$.

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