According to Ampere's circuital law,

$$\oint \vec{H} \cdot \vec{dL} = I.$$

$$\int_{\phi=0}^{2\pi} H_\phi r\, d\phi = I$$

$$H_\phi r [\phi]_0^{2\pi} = I$$

$$H_\phi r 2\pi = I$$

$$\Rightarrow \quad H_\phi = \frac{I}{2\pi r}$$

Hence $\vec{H}$ at Point P is given by

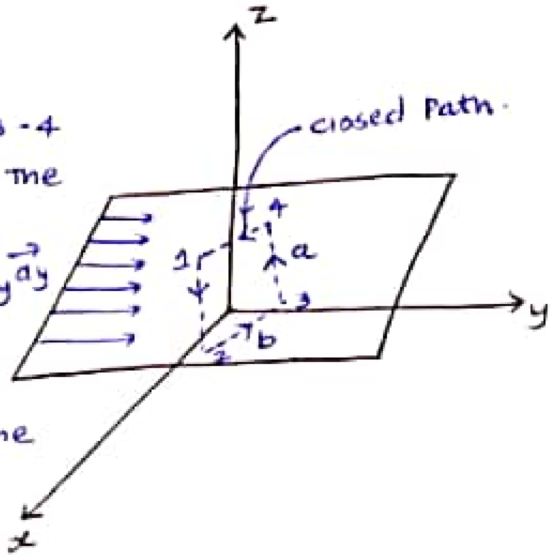
$$\boxed{\vec{H} = H_\phi \vec{a_\phi} = \frac{I}{2\pi r} \vec{a_\phi} \quad A/m}$$

$$\therefore \boxed{\vec{B} = \mu\vec{H} = \frac{\mu I}{2\pi r} \vec{a_\phi}}$$

$\vec{H}$ due to infinite sheet of current:

Consider an infinite sheet of current in the $z=0$ plane. The surface current density is $\vec{K}$. The current is flowing in Positive y direction hence

$$\vec{K} = K_y \vec{a_y}$$



Consider a closed path 1-2-3-4 as shown in fig. The width of the Path is 'b' while height is 'a'.

It is $\perp^r$ to the direction of current hence in xz plane.

The current flowing across the distance 'b' is given by $K_y b$

$$I_{enc} = K_y b \quad \text{---- (1)}$$

consider path 2-3 along which $\vec{dL} = dx\,\vec{a_x}$

$$\therefore \int_2^3 \vec{H}\cdot\vec{dL} = \int_2^3 (H_x\vec{a_x})\cdot(dx\,\vec{a_x}) = -H_x\int_2^3 dx = bH_x \quad \text{---} (4)$$

The path 2-3 is lying in $z<0$ region for which $\vec{H}$ is $-H_x\vec{a_x}$. And limits from 2 to 3, positive x to negative x hence effective sign of the integral is +ve

consider path 4-1 along which $\vec{dL} = dx\,\vec{a_x}$ and its in the region $z>0$ hence $\vec{H} = H_x\vec{a_x}$

$$\therefore \int_4^1 \vec{H}\cdot\vec{dL} = \int_4^1 (H_x\vec{a_x})\cdot(dx\,\vec{a_x}) = H_x\int_4^1 dx = bH_x \quad \text{---} (5)$$

$\therefore$ sub (4) and (5) in (3) we get

$$\oint \vec{H}\cdot\vec{dL} = bH_x + bH_x = 2bH_x \quad \text{---} (6)$$

usin (1) in eqv (6) we get

$$\oint \vec{H}\cdot\vec{dL} = 2bH_x = I_{enc} = K_y b$$

$$\therefore 2bH_x = K_y b$$

$$H_x = \frac{1}{2}K_y$$

Hence $\vec{H} = \frac{1}{2}K_y\vec{a_x}$ for $z>0$

$= -\frac{1}{2}K_y\vec{a_x}$ for $z<0$

In general for an infinite sheet of current density $\vec{K}$ A/m we can write

$$\vec{H} = \frac{1}{2}\vec{K}\times\vec{a_N}$$

where $\vec{a_N}$ = unit vector normal from the current sheet to the point at which $\vec{H}$ is to be obtained.

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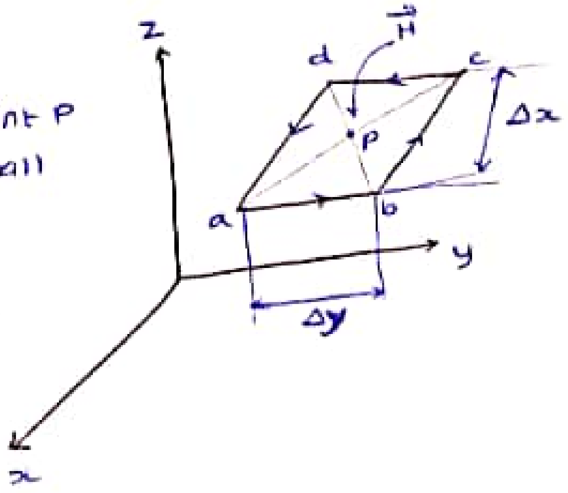
CURL:

Consider the differential surface element having sides Δx and Δy plane, as shown in figure below. The unknown current has produced $\vec{H}$ at the centre of the incremental closed path.



The total magnetic field at point P which is at the centre of the small rectangle is,

$$\vec{H} = H_{x0}\vec{a_x} + H_{y0}\vec{a_y} + H_{z0}\vec{a_z} \quad ---(1)$$

while the total current density is given by,

$$\vec{J} = J_x\vec{a_x} + J_y\vec{a_y} + J_z\vec{a_z} \quad ---(2)$$

To apply Ampere's circuital law to this closed path, let us evaluate the closed line integral of $\vec{H}$ about this path in the direction abcda. According to right hand thumb rule the current is in $\vec{a_z}$ direction.

Along path a-b,

$$\vec{H} = H_y\vec{a_y} \; \& \; \vec{dL} = \Delta y\,\vec{a_y}$$

$$\therefore \vec{H}\cdot\vec{dL} = H_y\,\Delta y \quad ----(3)$$

The intensity H_y along a-b can be expressed in terms of H_{y0} existing at P and the rate of change of H_y in the x-direction with x.

The distance in x direction of a-b from point P is $\left(\frac{\Delta x}{2}\right)$. Hence $\vec{H}\cdot\vec{dL}$ along a-b can be expressed as.

$$(\vec{H}\cdot\vec{dL})_{ab} = \left[H_{y0} + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x}{2}\right)\right]\Delta y \quad ---(4)$$

For path b-c, $\vec{H}$ is in $-\vec{a_x}$ direction hence $-H_x\vec{a_x}$ and $\vec{dL} = \Delta x\,\vec{a_x}$

$$\therefore \vec{H}\cdot\vec{dL} = -H_x\,\Delta x \quad ---(5)$$

Now H_x can be expressed interms of H_{x0} at Point P and rate of change of H_x in y direction and y

$$H_x = H_{x0} + \frac{\Delta y}{2} \frac{\partial H_x}{\partial y}$$

The distance of bc from P is $\Delta y/2$.

$$\therefore (\vec{H}.\vec{dL})_{b-c} = -\left[H_{x0} + \frac{\partial H_x}{\partial y} \frac{\Delta y}{2}\right] \Delta x \quad --- (6)$$

For Path c-d $\vec{H}$ is in $-\vec{a_y}$ direction hence $-H_y \vec{a_y}$ and $\vec{dL} = \Delta y \vec{a_y}$

$$\therefore \vec{H}.\vec{dL} = -H_y \Delta y \quad ---- (7)$$

But H_y can be expressed in terms of H_{y0} and rate of change of H_y in negative x direction. The distance of CD from point P is $(\Delta x/2)$ in negative x direction

$$\therefore H_y = H_{y0} - \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}$$

$$\therefore (\vec{H}.\vec{dL})_{c-d} = -\left[H_{y0} - \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}\right] \Delta y \quad --- (8)$$

For Path d-a, $\vec{H}$ is in $+\vec{a_x}$ direction hence $H_x \vec{a_x}$ and $\vec{dL} = \Delta x \vec{a_x}$

$$\therefore \vec{H}.\vec{dL} = H_x \Delta x \quad --- (9)$$

But H_x can be expressed interms of H_{x0} and rate of change of H_x in negative y direction. The distance of DA from Point P is $(\frac{\Delta y}{2})$ in negative y direction

$$\therefore H_x = \left[H_{x0} - \frac{\Delta y}{2} \frac{\partial H_x}{\partial y}\right] \cancel{\Delta x} . \cancel{---} (10)$$

$$\therefore (\vec{H}.\vec{dL})_{d-a} = \left[H_{x0} - \frac{\Delta y}{2} \frac{\partial H_x}{\partial y}\right] \Delta x \quad --- (10)$$

Total $\vec{H}\cdot\vec{dL}$ can be obtained by adding equations (4), (6), (8) and (10)

$$\therefore \vec{H}\cdot\vec{dL} = H_{y0}\Delta y + \frac{\Delta x \Delta y}{2}\frac{\partial H_y}{\partial x} - H_{x0}\Delta x - \frac{\Delta x \Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$- H_{y0}\Delta y + \frac{\Delta x \Delta y}{2}\frac{\partial H_y}{\partial x} + H_{x0}\Delta x - \frac{\Delta x \Delta y}{2}\frac{\partial H_x}{\partial y}.$$

$$\oint \vec{H}\cdot\vec{dL} = \Delta x \Delta y \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \quad \text{----- (11)}$$

According to Ampere's circuital law, this integral must be current enclosed by the differential element.

current enclosed = current density normal to closed path × Area of that closed path

$$I_{encl.} = J_z \Delta x \Delta y \quad \text{--- (12)}$$

where J_z = current density in $\vec{a_z}$ direction as the current enclosed is in $\vec{a_z}$ direction.

From eqn (11) & (12)

$$\oint \vec{H}\cdot\vec{dL} = I_{encl}.$$

$$\oint \vec{H}\cdot\vec{dL} = \Delta x \Delta y \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = J_z \Delta x \Delta y$$

$\div \Delta x \Delta y$

$$\frac{\oint \vec{H}\cdot\vec{dL}}{\Delta x \Delta y} = \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = J_z \quad \text{--- (13)}$$

This gives accurate result as the closed path shrinks to a point (ie) $\Delta x \Delta y$ area tends to zero.

$$\lim_{\Delta x \Delta y \to 0} \frac{\oint \vec{H}\cdot\vec{dL}}{\Delta x \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z \quad \text{--- (14)}$$

Considering incremental closed path in yz plane we get the current density normal to it i.e in y-direction so we can write,

$$\lim_{\Delta y \Delta z \to 0} \frac{\oint \vec{H} \cdot \vec{dL}}{\Delta y \, \Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x \quad \text{--- (15)}$$

and

$$\lim_{\Delta z \Delta x \to 0} \frac{\oint \vec{H} \cdot \vec{dL}}{\Delta z \, \Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y \quad \text{--- (16)}$$

In general we can write,

curl.

$$\lim_{\Delta S_N \to 0} \frac{\oint \vec{H} \cdot \vec{dL}}{\Delta S_N} = J_N \quad \text{--- (17)}$$

where J_N = current density normal to the surface ΔS

The term on left hand side of the equation is called curl $\vec{H}$. The ΔS_N is area enclosed by the closed line integral

The total $\vec{J}$ now can be obtained by adding eqn (14), (15) and (16)

$$\therefore \vec{J} = J_x \vec{a_x} + J_y \vec{a_y} + J_z \vec{a_z}$$

$$= \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] \vec{a_x} + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right] \vec{a_y} + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \vec{a_z}$$

$$\boxed{\vec{J} = \text{curl } \vec{H} = \nabla \times \vec{H}} \quad \text{--- (18)}$$

The curl $\vec{H}$ is indicated by $\nabla \times \vec{H}$ which is cross product of operator 'del' and $\vec{H}$

The equation (18) is called the point form of Ampere's circuital law

$$\boxed{\text{curl } \vec{H} = \nabla \times \vec{H} = \vec{J}}$$

∴ This is one of the Maxwell's equations.

The curl of a vector in the direction of the unit vector is the [illegible] of the line integral of the vector around a closed contour, as the enclosed area bounded by the [illegible] as the enclosed area decreases to zero

$\therefore$ This is one of the Maxwell's equations.

The curl of a vector in the direction of the unit vector is the ratio of the line integral of the vector around a closed contour, to the enclosed area bounded by the contour, as the enclosed area diminished to zero.

Properties of curl:

1. The curl of a vector is a vector quantity
2. $\nabla \times (\vec{A}+\vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$
3. $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 A$
4. The divergence of a curl is zero
   $\nabla \cdot (\nabla \times \vec{A}) = 0.$
5. The curl of a gradient of a vector is zero
   $\nabla \times \nabla V = 0.$

STROKE'S THEOREM:

Analogous to the divergence theorem in electrostatics, there exist stoke's theorem in magnetostatics. The stoke's theorem relates the line integral to a surface integral.

The stoke's theorem states that,

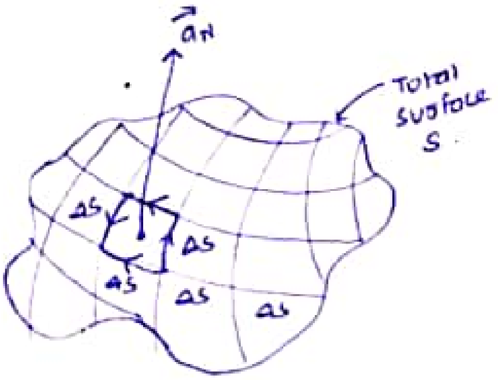
"The line integral of vector $\vec{A}$ around a closed path L is equal to the integral of curl of $\vec{A}$ over the open surface S enclosed by the closed path L".

The theorem is applicable only when $\vec{A}$ and $\nabla \times \vec{A}$ are continuous on the surface S.

$$\boxed{\oint_L \vec{H} \cdot \vec{dL} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}}$$

PROOF'S OF STROKE'S THEOREM:

Consider a surface S which is splitted into number of incremental surfaces. Each incremental surface is having area ΔS as shown in figure.



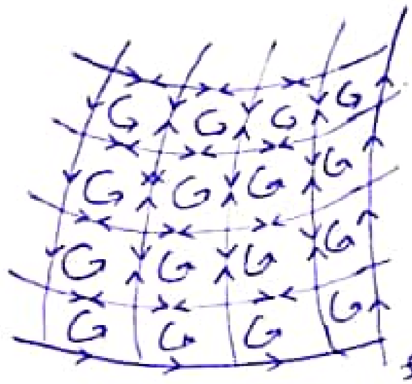


fig (b)

Applying by definition of the curl to any of these incremental surfaces we can write

$$(\nabla \times \vec{H})_N = \frac{\oint \vec{H} \cdot \vec{dL}_{\Delta s}}{\Delta s} \quad \text{--- ①}$$

where

$N \rightarrow$ Normal to Δs according to right hand rule

$dL_{\Delta s} \rightarrow$ Perimeter of the incremental surface Δs

Now the curl of $\vec{H}$ in the normal direction is the dot product of curl of $\vec{H}$ with $\vec{a_N}$, where $\vec{a_N}$ is unit vector, normal to the surface Δs, according to right hand rule.

$$\therefore (\nabla \times \vec{H})_N = (\nabla \times \vec{H}) \cdot \vec{a}_N$$

$$\therefore \oint \vec{H} \cdot \vec{dL}_{\Delta s} = (\nabla \times \vec{H}) \cdot \vec{a}_N \, \Delta s$$

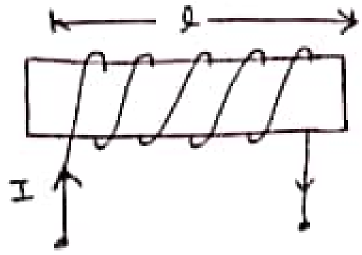
$$\therefore \oint \vec{H} \cdot \vec{dL}_{\Delta s} = (\nabla \times \vec{H}) \cdot \vec{\Delta s}$$

To obtain total curl for every incremental surface, add the closed line integrals for each Δs. From fig (b), it can be seen that at a common boundary b/w the two incremental surfaces, the line integral is getting cancelled as the boundary is getting traced in two opposite directions.

INDUCTANCE OF A SOLENOID:

Consider a solenoid of N Turns as shown in fig

Let the current flowing thro' the solenoid be I Amps. Let the length of the solenoid be l and the cross-section area be A.



~~Elect~~ Magnetic field intensity H inside the solenoid is given by,

$$H = \frac{NI}{l} \; (A/m) \; \text{---} \; (1)$$

Total flux linkage is given by

Total flux linkage $= N\phi = N(B)(A)$

$$\left[\because \phi = BA \qquad Wb = \frac{Wb}{m^2} \times m^2\right]$$

$$N\phi = NBA$$

But $B = \mu H$

$$\therefore N\phi = N(\mu H)A$$

$$= \mu NHA$$

$$= \mu N\left[\frac{NI}{l}\right]A \Rightarrow N\phi = \frac{\mu N^2 IA}{l}$$

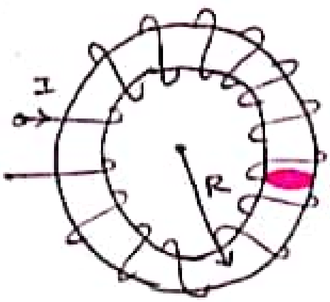
Inductance of a solenoid is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}} = \frac{\mu N^2 IA/l}{I}$$

$$\boxed{L = \frac{\mu N^2 A}{l}}$$

INDUCTANCE OF A TOROID:

Consider a toroid ring with N Turns and carrying current I. Let the radius of the Toroid be R as shown in fig.



The magnetic flux density inside a toroidal ring is given by

$$B = \frac{\mu NI}{2\pi R} \; \text{---} \; (1)$$

$$\left[\because \quad B = \mu H = \mu \cdot \left(\frac{NI}{l}\right) \rightarrow \text{Perimeter of circle}\right.$$

$$= \mu\left(\frac{NI}{2\pi R}\right)$$

Let $\vec{B} = \frac{NI}{2\pi r}\vec{a}_\varphi$ (T)

The total magnetic flux is given by

$$\varphi = \int_S \vec{B}\cdot\vec{ds}$$

Now $\vec{ds} = dr\,dz\,\vec{a}_\varphi$ [from cylindrical co-ordinate system].

$$\therefore \varphi = \int_{z=0}^{z=d}\int_{r=a}^{r=b} \frac{NI}{2\pi r}\vec{a}_\varphi \cdot dr\,dz\,\vec{a}_\varphi$$

$$= \frac{NI}{2\pi}\int_{z=0}^{z=d} dz \int_{r=a}^{r=b} \frac{1}{r}\,dr$$

$$= \frac{NI}{2\pi}\left[z\right]_0^d \left[\ln r\right]_a^b$$

$$\boxed{\varphi = \frac{NI}{2\pi} d \ln\left(\frac{b}{a}\right)} \quad \text{--- (3)}$$

The inductance of a co-axial cable is given by

$$L = \frac{\text{Total flux linkage}}{\text{Total current}}$$

$$\boxed{L = \frac{\frac{NId}{2\pi}\ln\left(\frac{b}{a}\right)}{I} = \frac{Nd}{2\pi}\ln\left(\frac{b}{a}\right)\ \text{H}}$$

The Inductance of a co-axial cable may be expressed per unit length as

$$\boxed{\frac{L}{d} = \frac{N}{2\pi}\ln\left(\frac{b}{a}\right)\ \text{H/m}.}$$

MAGNETIC ENERGY - ENERGY STORED IN A MAGNETIC FIELD

Energy stored in a inductor is given by

$$W_m = \frac{1}{2}LI^2$$