Thus total flux intensity H can be obtained by integrating $\vec{dH}$ over the entire length of the conductor.

$$\therefore \vec{H} = \int_{z=-\infty}^{\infty} \vec{dH} = \int_{z=-\infty}^{\infty} \frac{I r dz \vec{a_\phi}}{4\pi (r^2+z^2)^{3/2}}$$

Put $z = r\tan\theta$ ← can be obtained by using $z = r\tan\theta$.

$z^2 = r^2\tan^2\theta$

and $dz = r\sec^2\theta \, d\theta$, $z=-\infty,\ \theta=-\pi/2$ & $z=+\infty,\ \theta=+\pi/2$

$$\therefore \vec{H} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \frac{I r \, r\sec^2\theta \, d\theta \, \vec{a_\phi}}{4\pi (r^2+r^2\tan^2\theta)^{3/2}}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{I r^2 \sec^2\theta \, d\theta \, \vec{a_\phi}}{4\pi r^3 (\sec^3\theta)} \qquad \left[\because (r^2+r^2\tan^2\theta)^{3/2} = [r^2(1+\tan^2\theta)]^{3/2} = r^3\sec^3\theta\right] \quad 1+\tan^2\theta = \sec^2\theta$$

$$\vec{H} = \int_{-\pi/2}^{\pi/2} \frac{I}{4\pi r} \cdot \frac{1}{\sec\theta} \cdot d\theta \, \vec{a_\phi} \Rightarrow \frac{I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \vec{a_\phi}$$

$$\vec{H} = \frac{I}{4\pi r} [\sin\theta]_{-\pi/2}^{\pi/2} \vec{a_\phi} = \frac{I}{4\pi r}\left[\sin\pi/2 - \sin(-\pi/2)\right] \vec{a_\phi}$$

$$\vec{H} = \frac{I}{4\pi r}[1-(-1)]\vec{a_\phi} = \frac{2I}{4\pi r}\vec{a_\phi}$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\vec{a_\phi}} \ A/m \qquad \boxed{\vec{B} = \mu\vec{H} = \frac{\mu I}{2\pi r}\vec{a_\phi} \ Wb/m^2}$$

AMPERE'S CIRCUITAL LAW:

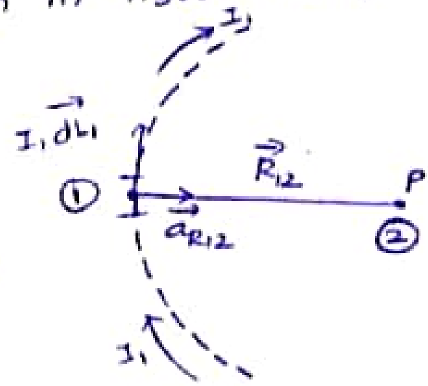
In electrostatics, the Gauss's law is useful to obtain the $\vec{E}$ in case of complex problems. Similarly in the magneto-statics, the complex problems can be solved using a law called Ampere's circuital law (or) Ampere's work law.

The entire conductor is made up of all such differential elements. Hence to obtain total magnetic field intensity $\vec{H}$, the equation ⑤ takes the integral form as,

$$\vec{H} = \oint \frac{I \, \vec{dL} \times \vec{a_R}}{4\pi R^2} \quad \text{----} \quad (7)$$

The closed line integral is required to ensure that all the current elements are considered. This is because the current can flow only in the closed path, provided by the closed circuit.

If the current element is considered at point 1 and point P at point 2, as shown in figure then,



$$\vec{dH_2} = \frac{I_1 \, \vec{dL_1} \times \vec{a_{R12}}}{4\pi R_{12}^2} \text{ A/m.}$$

$I_1 \rightarrow$ current flowing thro' dL_1 at point 1

$\vec{dL_1} \rightarrow$ Differential vector length at point 1

$\vec{a_{R12}} \rightarrow$ unit vector in the direction from element at point 1 to the point P at point 2

$$\vec{a_{R12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{R_{12}}}{R_{12}}$$

$$\boxed{\therefore \; H = \oint \frac{I_1 \vec{dL_1} \times \vec{a_{R12}}}{4\pi R_{12}^2} \text{ A/m}} \quad \text{---} \quad (8)$$

This is called Integral form of Biot-Savart Law.

The Biot-Savarts law states that

The magnetic field intensity $\vec{dH}$ produced at a point P due to a differential current element IdL is,

1. Proportional to the product of current I and differential length dL.

2. The sine of the angle b/w the element and the line joining point P to the element

3. And inversely proportional to the square of the distance R b/w point P and the element.

Mathematically, the Biot-Savarts law can be stated as,

$$\vec{dH} \propto \frac{I dL \sin\theta}{R^2} \quad ---(1)$$

$$\vec{dH} = \frac{k I dL \sin\theta}{R^2} \quad ----(2)$$

$k \rightarrow$ constant of proportionality

$k = \frac{1}{4\pi}$

$$\therefore \vec{dH} = \frac{I dL \sin\theta}{4\pi R^2} \quad ----(3)$$

Let dL = Magnitude of vector length $\vec{dL}$ and

$\vec{a_R}$ = unit vector in the direction from differential current element to point P.

Then from rule of cross product

$$\vec{dL} \times \vec{a_R} = dL\, |\vec{a_R}| \sin\theta = dL \sin\theta \quad ---(4)$$

Using (4) in (3) we get

$$\vec{dH} = \frac{I\, \vec{dL} \times \vec{a_R}}{4\pi R^2} \text{ A/m} \quad ---(5)$$

But $\vec{a_R} = \frac{\vec{R}}{|\vec{R}|} = \frac{\vec{R}}{R}$ $\therefore$ $\boxed{\vec{dH} = \frac{I\, \vec{dL} \times \vec{R}}{4\pi R^3} \text{ A/m}}$ ---(6)

Equations (5) and (6) is the mathematical form of Biot-Savarts law.

For free space

Permability is denoted as $\mu_0 = 4\pi \times 10^{-7}$ H/m

For any other region

a relative permability is specified as μ_r

and $\mu = \mu_0 \mu_r$.

The $\vec{B}$ and $\vec{H}$ are related as

$$\boxed{\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}}$$

For free space

$$\boxed{\vec{B} = \mu_0 \vec{H}}$$

For nonmagnetic media

$\mu_r = 0$

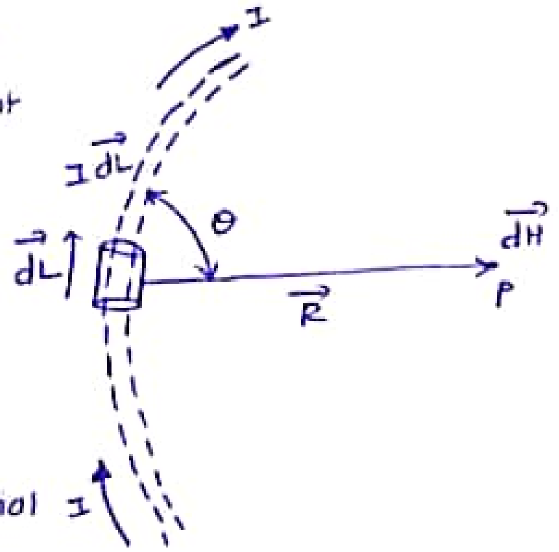
For magnetic materials

$\mu_r > 1$.

BIOT-SAVART LAW:

Consider a conductor carrying a direct current I and a steady magnetic field produced around it. The Biot-savart law allows us to obtain the differential magnetic field intensity $d\vec{H}$, produced at point P, due to a differential current element $I\,dL$.

Consider a differential length dL hence the differential current element is $I\,dL$. This is a very small part of the current carrying conductor. The point P is at a distance R from the differential current element. The θ is the angle b/w the differential current element and the line joining point p to the differential current element.



The magnetic lines of force i.e magnetic flux lines always form a closed loop and exist in the form of concentric circles, around a current carrying conductor. The total number of magnetic lines of force is called a magnetic flux denoted as ϕ. It is measured in weber (Wb). one Wb means 10^8 lines of force.

MAGNETIC FIELD INTENSITY: ($\vec{H}$)

The quantitative measure of strongness or weakness of the magnetic field is given by magnetic field intensity or magnetic field strength. The magnetic field intensity at any point in the magnetic ~~field~~ field is defined as the force experienced by a unit north poles of one weber strength, when placed at that point.

The magnetic flux lines are measured in webers (Wb) while magnetic field intensity is measured in Newtons/weber [N/Wb] or amperes per meter [A/m]. or ampere Turns/meter [AT/m]. It is denoted by $\vec{H}$.

MAGNETIC FLUX DENSITY: ($\vec{B}$)

The Total magnetic lines of ~~force~~ i.e magnetic flux crossing a unit area in a plane at right angles to the direction of flux is called magnetic flux density. It is denoted by $\vec{B}$. It is measured in wb/m^2 which is also called Tesla (T).

RELATION BETWEEN $\vec{B}$ & $\vec{H}$.

In magnetostatics, the $\vec{B}$ and $\vec{H}$ are related to each other thro' the property of the region in which current carrying conductor is placed. It is called Permeability denoted as μ.

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The ampere's circuital law states that,

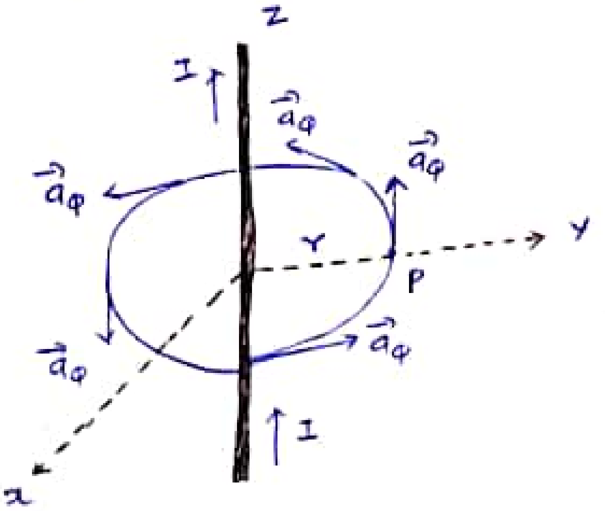
The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path

The mathematical representation of Ampere's circuital law is,

$$\boxed{\oint \vec{H}\cdot\vec{dL} = I}$$

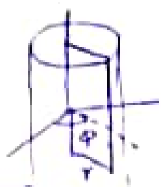
PROOF FOR AMPERE'S CIRCUITAL LAW:

Consider a long straight conductor carrying direct current I placed along z-axis as shown in figure below.



Consider a closed circular path of radius r which encloses the straight conductor carrying direct current I. The point P is at a $\perp^r$ distance r from the conductor. Consider $\vec{dL}$ at point P which is in $\vec{a}_\phi$ direction, tangential to circular path at point P.

$$\therefore \vec{dL} = r\,d\phi\,\vec{a}_\phi \quad \text{---}(1)$$



While $\vec{H}$ obtained at point P, from Biot-Savart's law due to infinitely long conductor is

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_\phi \quad [\text{from Page No } ⑤ \text{ of unit } \overline{IV}] \quad \text{---}(2)$$

$$\therefore \vec{H}\cdot\vec{dL} = \frac{I}{2\pi r}\vec{a}_\phi \cdot r\,d\phi\,\vec{a}_\phi$$

$$\vec{H}\cdot\vec{dL} = \frac{I\,r\,d\phi}{2\pi r}$$

$$\vec{H}\cdot\vec{dL} = \frac{I\,d\phi}{2\pi} \quad \text{---} (3)$$

Integrating (3) over the entire closed path,

$$\oint \vec{H}\cdot\vec{dL} = \int_{\phi=0}^{2\pi} \frac{I\,d\phi}{2\pi} = \frac{I}{2\pi}\left[\phi\right]_0^{2\pi} = I$$

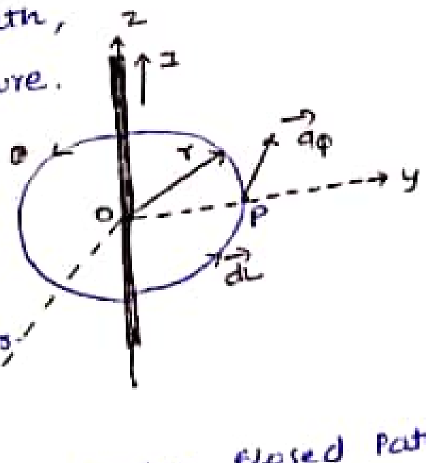
= current carried by conductor

This proves that the integral $\vec{H}\cdot\vec{dL}$ along closed path gives direct current enclosed by that closed path.

APPLICATION OF AMPERE'S CIRCUITAL LAW:

$\vec{H}$ due to infinitely (straight conductor)

Consider an infinitely long straight ~~line~~ conductor placed along z-axis, carrying a direct current I as shown in the figure. Consider the Amperian closed path, enclosing the conductor as shown in figure. Consider point P on the closed path at which $\vec{H}$ is to be obtained. The radius of the path is r and hence P is at a $\perp^r$ distance from the conductor.



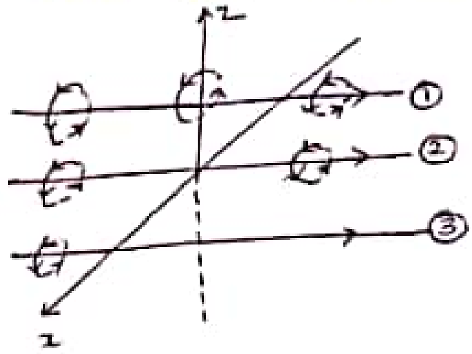
The magnitude of $\vec{H}$ depends on r and the direction is always tangential to the closed path i.e $\vec{a}_\phi$. So $\vec{H}$ has only component in $\vec{a}_\phi$ direction say H_ϕ.

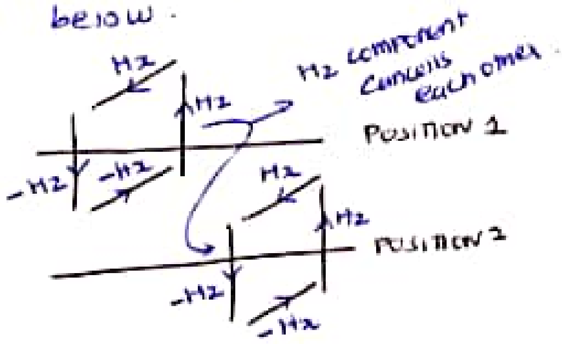
Consider elementary length $\vec{dL}$ at a point P and in cylindrical co-ordinates it is $r\,d\phi$ in $\vec{a}_\phi$ direction.

$$\therefore \vec{H} = H_\phi \vec{a}_\phi \quad \text{and} \quad \vec{dL} = r\,d\phi\,\vec{a}_\phi$$

$$\therefore \vec{H}\cdot\vec{dL} = H_\phi\vec{a}_\phi \cdot r\,d\phi\,\vec{a}_\phi = H_\phi\, r\,d\phi$$

consider the magnetic lines of force due to the current in $\vec{a_y}$ direction, according to the right hand thumb rule. These are shown in figure below.





It is clear that in b/w two very closely spaced conductors, the components of $\vec{H}$ in z direction are oppositely directed [$-H_z$ for position 1 and $+H_z$ for position 2 b/w the two opposite positions]. All such components cancel eachother and hence $\vec{H}$ cannot have any component in z-direction.

As current is flowing in y direction, $\vec{H}$ cannot have component in y direction.

component in x direction.

So $\vec{H}$ has only component in x direction.

$$\left.\begin{aligned} \vec{H} &= H_x \vec{a_x} \quad \text{for} \quad z > 0 \\ &= -H_x \vec{a_x} \quad \text{for} \quad z < 0 \end{aligned}\right\} \text{---- (2)}$$

Applying Ampere's circuital law

$$\oint \vec{H} \cdot \vec{dL} = I_{enc} \text{ ---- (3)} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 [\vec{H} \cdot \vec{dL}]$$

Evaluate the integral along the path 1-2-3-4-1.

For path 1-2, $\vec{dL} = dz\, \vec{a_z}$

3-4, $\vec{dL} = -dz\, \vec{a_z}$

But $\vec{H}$ is in x-direction while $\vec{a_x} \cdot \vec{a_z} = 0$.

Hence along the paths 1-2 and 3-4, the integral

$$\oint \vec{H} \cdot \vec{dL} = 0$$