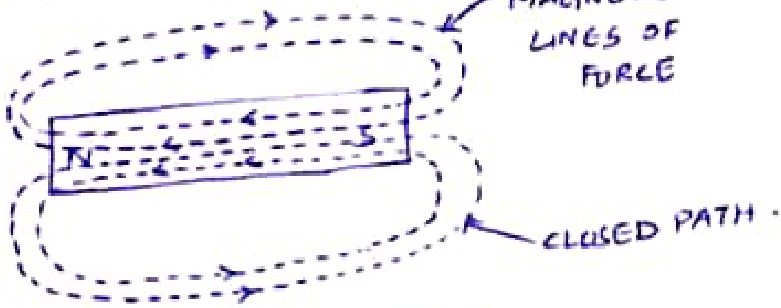
MAGNETIC FIELD & ITS PROPERTIES

Consider a Permanent magnet, It has two Poles North (N) and South (S). The region around a magnet within which the influence of the magnet can be experienced is called magnetic field.

Such a field is represented by imaginary lines around the magnet which are called magnetic lines of force. These lines of force are also called magnetic lines of flux or magnetic flux lines.

An important difference b/w electric flux lines and magnetic flux lines can be observed here. In case of electric flux, the flux lines originate from an isolated positive charge and diverge to terminate at infinity. while for a -ve charge, electric flux lines converge on a charge, starting from infinity. But in case of magnetic flux, the Poles exists in pairs only.

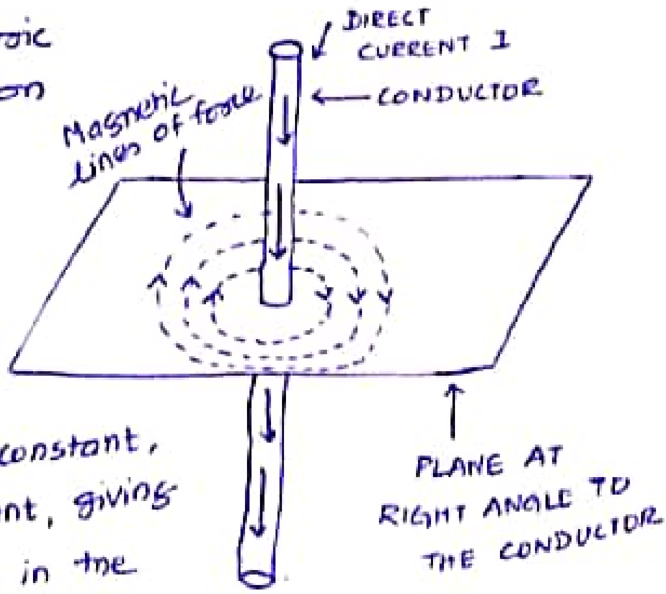
Hence every magnetic flux line starting from North Pole must end at south Pole and complete the path from south to north internal to the magnet.



MAGNETIC FIELD DUE TO CURRENT CARRYING CONDUCTOR:

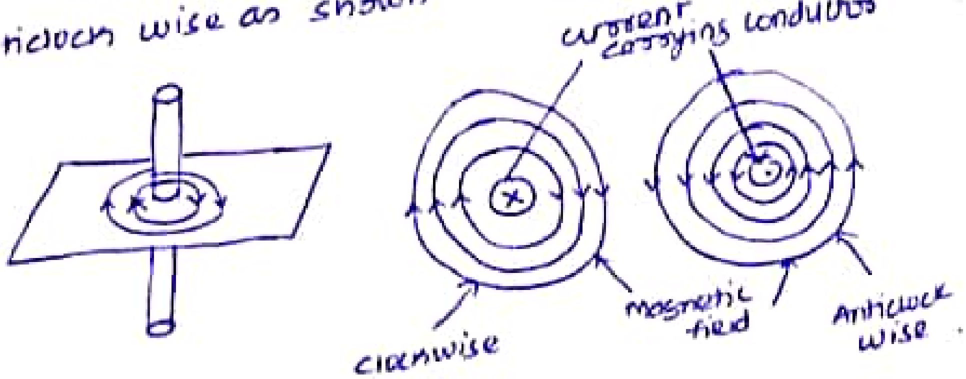
When a straight conductor carries a direct current, it produces a magnetic field around it, all along its length. The lines of force in such a case are in the form of concentric circles in the planes at right angles to the conductor.

The direction of concentric circles around depends on the direction of current thro' the conductor. As long as the current is constant and current is time independent, magnetic lines of force are also constant, static and time independent, giving a steady magnetic field in the space around the conductor.



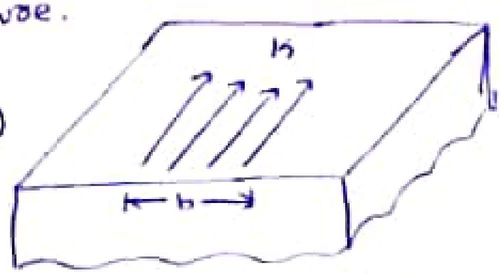
A right hand thumb rule is used to determine the direction of magnetic field around a conductor carrying a direct current. It states that, hold the current carrying conductor in right hand such that thumb pointing in the direction of current and $\parallel^{al}$ to the conductor, then curled fingers point in the direction of the magnetic lines of flux around it.

The cross indicates that the current direction is going ~~up~~ into the plane of the paper away from the observer. The dot indicates that the current direction is coming out of the plane of the paper, coming towards the observer. Using right hand thumb rule, the direction of magnetic flux around such a conductor is either clockwise (or) anticlockwise as shown in figure.



BIOT-SAVART LAW INTERMS OF DISTRIBUTED SOURCES

Consider a surface carrying a uniform current over its surface as shown in figure. Then the surface current density is denoted as $\vec{K}$ and is measured in (A/m)



Thus for uniform current density, current I in any width b is given by $I = Kb$, where width b is $\perp^r$ to the direction of current flow.

Thus if ds is the differential surface area considered of a surface having current density $\vec{K}$ then

$$I\,\vec{dL} = \vec{K}\,ds \quad \text{---- (1)}$$

If the current density in a volume of a given conductor is $\vec{J}$ measured in A/m^2 then the differential volume dv we can write

$$I\,\vec{dL} = \vec{J}\,dv \quad \text{--- (2)}$$

Hence biot-savart's law can be expressed for surface current considering $\vec{K}\,ds$ while for volume current considering $\vec{J}\,dv$.

$$\boxed{\begin{aligned} \therefore\ \vec{H} &= \oint_S \frac{\vec{K} \times \vec{a_R}\, ds}{4\pi R^2} \ A/m \\ \vec{H} &= \oint_{vol} \frac{\vec{J} \times \vec{a_R}\, dv}{4\pi R^2} \ A/m \end{aligned}} \quad \text{---- (3)}$$

The Biot-savart's law is also called Ampere's law for the current element.

$\vec{H}$ due to infinitely Long straight conductor:

Consider an infinitely long straight conductor, along z-axis the current passing through the conductor is a direct current of I Amp. The field intensity $\vec{H}$ at a point P is to be calculated, which is at a distance 'r' from the z-axis.

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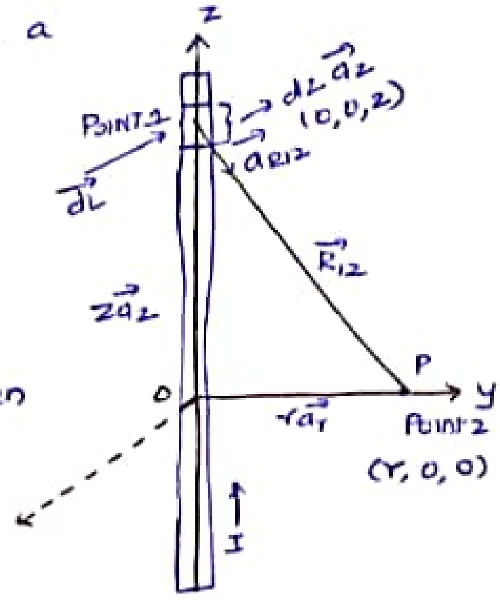
Consider small differential element at point 1, along the z-axis at a distance of z from origin.

$$\boxed{\therefore\ I\,\vec{dL} = I\,dz\,\vec{a_z}} \quad \text{---}\ (1)$$



The distance vector joining point 1 to point 2 is $\vec{R_{12}}$ and can be written as

$$\vec{R_{12}} = (r-0)\vec{a_r} + (0-0)\vec{a_\phi} + (0-z)\vec{a_z}$$

$$= r\vec{a_r} + 0\vec{a_\phi} - z\vec{a_z}$$

$$\boxed{\therefore\ \vec{R_{12}} = r\vec{a_r} - z\vec{a_z}} \quad \text{---}\ (2)$$

$$\vec{a_{R12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{-z\vec{a_z} + r\vec{a_r}}{\sqrt{r^2+z^2}} = \frac{r\vec{a_r} - z\vec{a_z}}{\sqrt{r^2+z^2}}$$

$$\vec{dL} \times \vec{a_{R12}} = \begin{bmatrix} \vec{a_r} & \vec{a_\phi} & \vec{a_z} \\ 0 & 0 & dz \\ r & 0 & -z \end{bmatrix} = r\,dz\,\vec{a_\phi}$$

Note: while taking cross product, $|\vec{R_{12}}|$ is neglected for convenience and must be considered for further calculations.

$$\therefore\ I\,\vec{dL} \times \vec{a_{R12}} = \frac{I r\,dz\,\vec{a_\phi}}{\sqrt{r^2+z^2}}$$

According to Biot-Savart's law, $\vec{dH}$ at point 2 is

$$\frac{I\,\vec{dl} \times \vec{a_{R12}}}{4\pi R^2}$$

$$\vec{dH} = \frac{I\,\vec{dL} \times \vec{a_{R12}}}{4\pi R_{12}^2} = \frac{I r\,dz\,\vec{a_\phi}}{\sqrt{r^2+z^2}} \cdot \frac{1}{4\pi\left(\sqrt{r^2+z^2}\right)^2}$$

$$\frac{I\,dz\,\vec{a_z}}{4\pi(r^2+z^2)} \times \frac{r\vec{a_r} - z\vec{a_z}}{\sqrt{r^2+z^2}}$$

$$\vec{dH} = \frac{I r\,dz\,\vec{a_\phi}}{4\pi(r^2+z^2)^{3/2}}$$

$a_z \times a_r = a_\phi$ $\quad$ $a_z \times a_z = 0$

$$\boxed{dH = \frac{I r\,dz}{4\pi(r^2+z^2)^{3/2}}\,a_\phi}$$

Thus total flux intensity H can be obtained by integrating $\vec{dH}$ over the entire length of the conductor. (5)

$$\therefore \vec{H} = \int_{z=-\infty}^{\infty} \vec{dH} = \int_{z=-\infty}^{\infty} \frac{I r dz \vec{a_\phi}}{4\pi (r^2+z^2)^{3/2}}$$

Put $z = r\tan\theta$ ← can be obtained by using $z = r\tan\theta$.

$z^2 = r^2\tan^2\theta$

and $dz = r\sec^2\theta \, d\theta$, $z=-\infty$, $\theta = -\pi/2$ & $z=+\infty$, $\theta = +\pi/2$

$$\therefore \vec{H} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \frac{I r \, r\sec^2\theta \, d\theta \, \vec{a_\phi}}{4\pi (r^2+r^2\tan^2\theta)^{3/2}}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{I r^2 \sec^2\theta \, d\theta \, \vec{a_\phi}}{4\pi r^3 (\sec^3\theta)}$$

$\left[\because (r^2+r^2\tan^2\theta)^{3/2} = [r^2(1+\tan^2\theta)]^{3/2} = r^3\sec^3\theta\right]$ $1+\tan^2\theta = \sec^2\theta$.

$$\vec{H} = \int_{-\pi/2}^{\pi/2} \frac{I}{4\pi r} \cdot \frac{1}{\sec\theta} \cdot d\theta \, \vec{a_\phi} \Rightarrow \frac{I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \vec{a_\phi}$$

$$\vec{H} = \frac{I}{4\pi r} [\sin\theta]_{-\pi/2}^{\pi/2} \vec{a_\phi} = \frac{I}{4\pi r}\left[\sin \pi/2 - \sin(-\pi/2)\right] \vec{a_\phi}$$

$$\vec{H} = \frac{I}{4\pi r}[1-(-1)]\vec{a_\phi} = \frac{2I}{4\pi r}\vec{a_\phi}$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\vec{a_\phi}} \text{ A/m} \qquad \boxed{\vec{B} = \mu\vec{H} = \frac{\mu I}{2\pi r}\vec{a_\phi} \text{ Wb/m}^2}$$

AMPERE'S CIRCUITAL LAW:

In electrostatics, the Gauss's law is useful to obtain the $\vec{E}$ in case of complex problems. Similarly in the magneto-statics, the complex problems can be solved using a law called Ampere's circuital law (or) Ampere's work law.