STRESSES IN THIN WALLED PRESSURE VESSEL

* Thin-walled pressure vessels provide an important application of the analysis of plane stress.

* Pressure vessels are closed structures containing liquids or gases under pressure. Familiar examples include tanks, pipes and pressurized cabins in aircraft and space vehicles.

* When pressure vessels have walls that are thin in comparison to their overall dimension, they are included within a more general ~~category~~ category known as shell structures.

* The term thin walled is not precise, but as a general rule, pressure vessels are considered to be thin-walled when the ratio of radius r to wall thickness is greater than 10.

* Here our analysis of stresses in thin-walled pressure vessels will be limited to the two types of vessels mostly

Scanned with CamScanner

frequently encountered:

(i) cylindrical pressure vessels

(ii) spherical pressure vessels.

CYLINDRICAL PRESSURE VESSEL

Consider a cylindrical vessel of inner radius r and wall thickness t containing a fluid under pressure.

We propose to determine the stresses exerted on a small element of wall with sides respectively parallel and perpendicular to the axis of the cylinder.

Because of the symmetry of the vessel and its loading, it is clear that no shearing stress on the element. The normal stresses ~~are~~ σ_1 and σ_2 shown in figure 7.47 are therefore principal stresses.

Because of their directions, the stress σ_1 is called the circumferential stress or the hoop stress.

Scanned with CamScanner

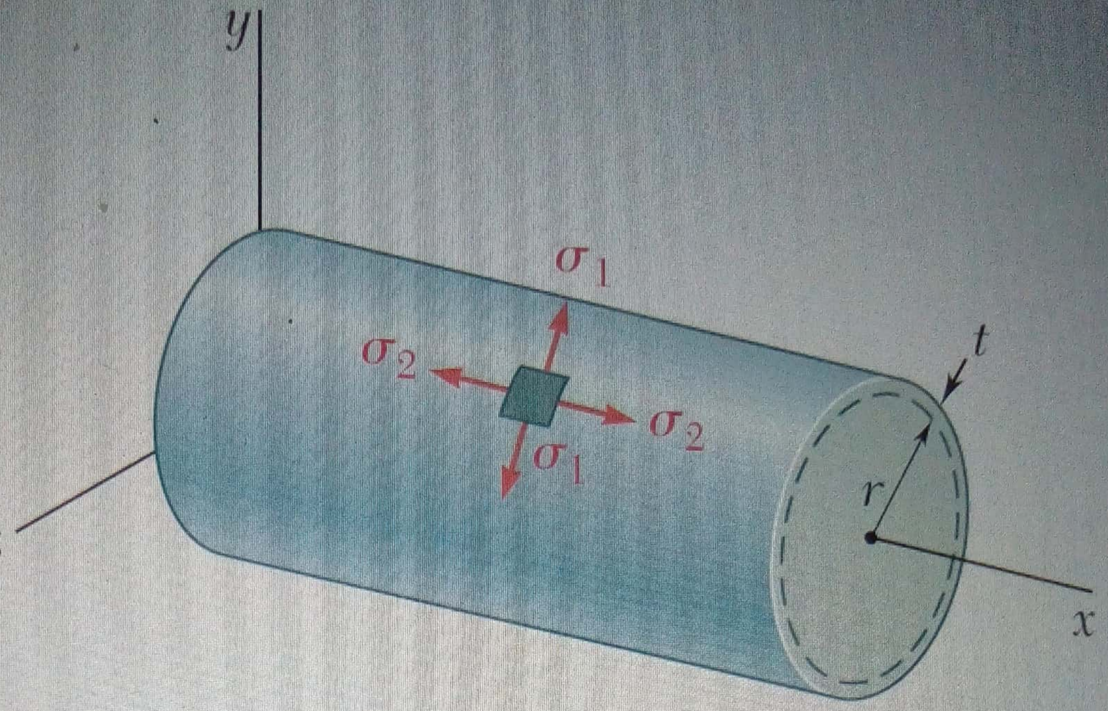

Fig. 7.47 Pressurized cylindrical vessel.

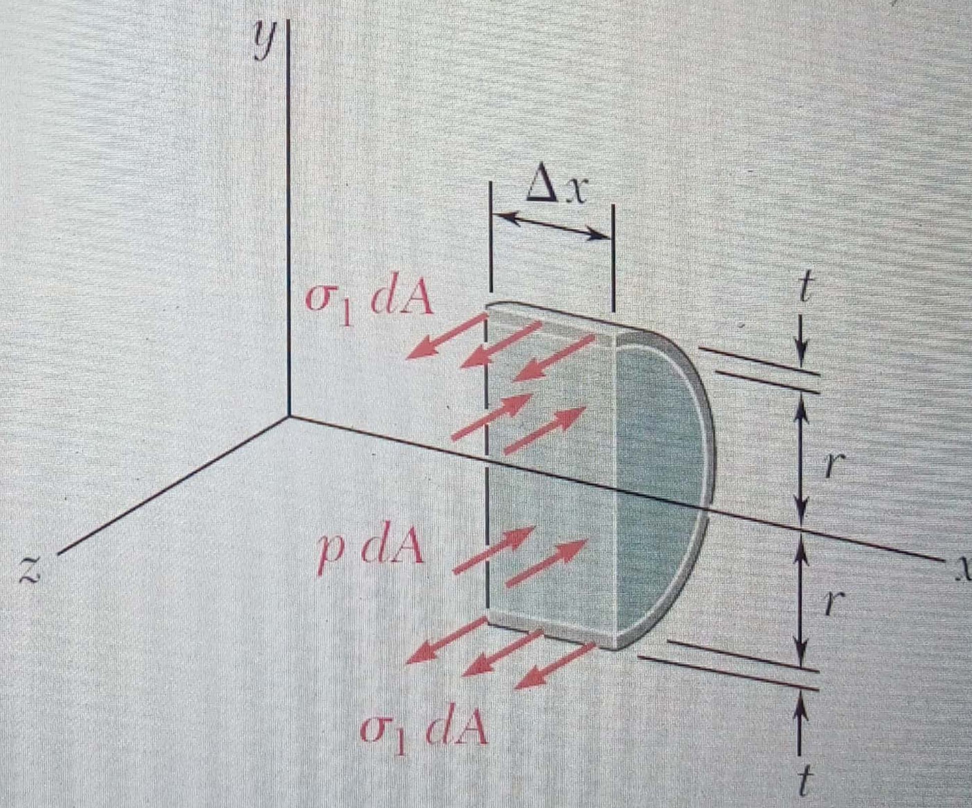

Fig. 7.48 Free body to determine hoop stress.

Page No.:
Date: / /

and the stress σ_2 is called the longitudinal stress or the axial stress.

Each of these stresses can be calculated from equilibrium by using appropriate free body diagrams.

In order to determine the hoop stress σ_1, we detach a portion of the vessel and its contents bounded by the xy plane and by two planes parallel to the YZ plane at a distance Δx from each other (figure 7.48)

Writing the equilibrium equation $\Sigma F_z = 0$, we have

$$\sigma_1(2t\,\Delta x) - p(2r\cdot\Delta x) = 0$$

$$\boxed{\therefore \sigma_1 = \frac{pr}{t}} \text{ --- } (A)$$

To determine the longitudinal stress σ_2, we now pass a section perpendicular to the x-axis and consider the free body consisting of the portion of the vessel and its contents located to the left of the section (fig 7.49)

Scanned with CamScanner

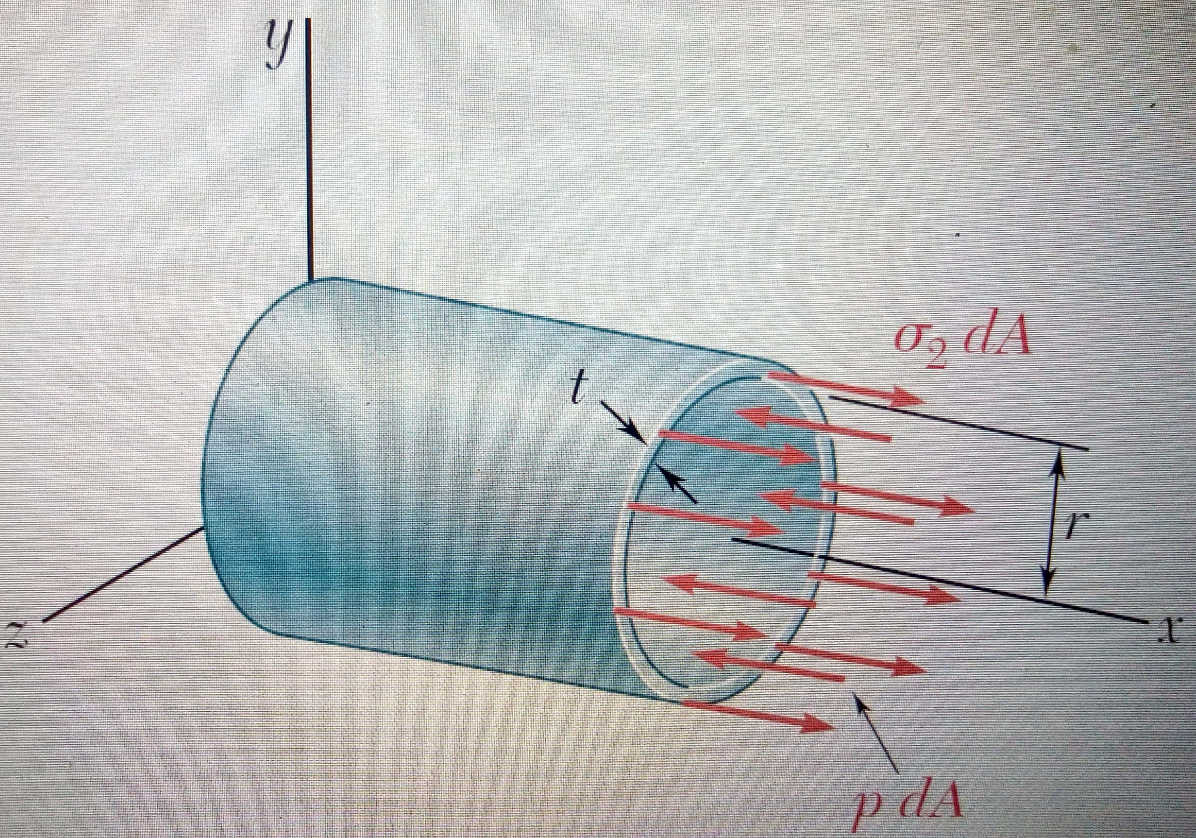

Fig. 7.49 Free body to determine longitudinal stress.

Writing the equilibrium equation

$\Sigma F_x = 0$

$\sigma_2 (2\pi r t) - p(\pi r^2) = 0$

$$\boxed{\therefore \sigma_2 = \frac{pr}{2t}} \longrightarrow (B)$$

From eqn (A) & (B), we observe that the hoop stress σ_1 is twice as large as the longitudinal stress σ_2

$$\boxed{\sigma_1 = 2\sigma_2}$$

Spherical Pressure vessels

* A sphere is the theoretically ideal shape for a vessel that resists internal pressure.

* To determine the stresses in a spherical vessel, let us cut through the sphere on a vertical diametral plane (fig 8-3a) and isolate half of the shell and its fluid content

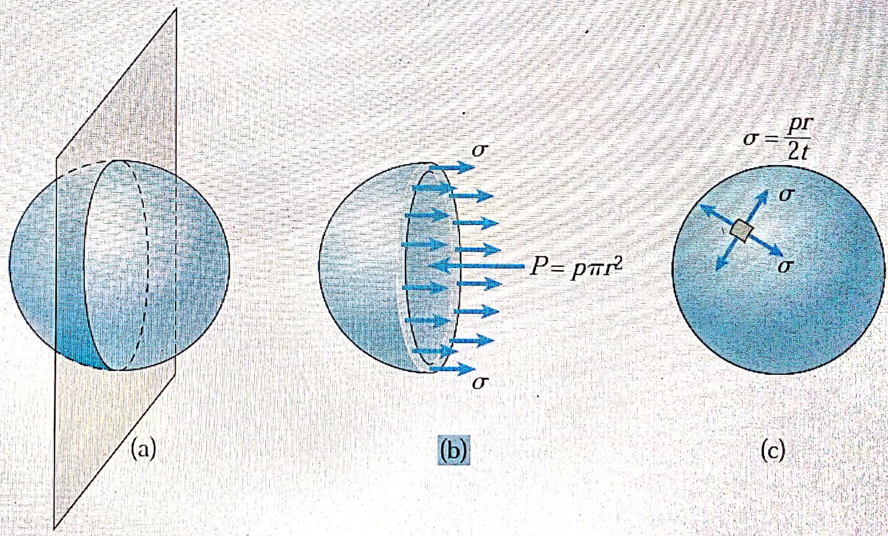

FIG. 8-3 Tensile stresses σ in the wall of a spherical pressure vessel

Page No.:
Date:

as a single free body (fig 8-3b)

The ~~fo~~ stresses acting on the free body are

1. the tensile stresses σ in the wall of the vessel.
2. the fluid pressure

fluid pressure acts horizontally against the plane circular area of fluid remaining inside the hemisphere. Since pressure is uniform, the resultant pressure force P ~~is~~

$$\boxed{P = p \cdot \pi r^2} \longleftarrow (A_1)$$

Because of the symmetry of the vessel and its loading, the tensile stress σ is uniform around the circumference

∴ horizontal force

$$\boxed{F = \sigma (2\pi r \cdot t)} \longleftarrow (B_1)$$

writing equilibrium eqn from (A_1) & (B_1)

we have $p \cdot \pi r^2 = \sigma \cdot (2\pi r) \cdot t$

$$\boxed{\therefore \sigma = \frac{pr}{2t}}$$

This is the expression for tensile stresses in the wall of a spherical shell

From symmetry of a spherical shell, we obtain the same equation for the tensile stresses when we cut a plane through the centre of the sphere in any direction whatsoever.

Thus, we reach the following conclusion.

The wall of a pressurised spherical vessel is subjected to uniform tensile stresses σ in all directions.