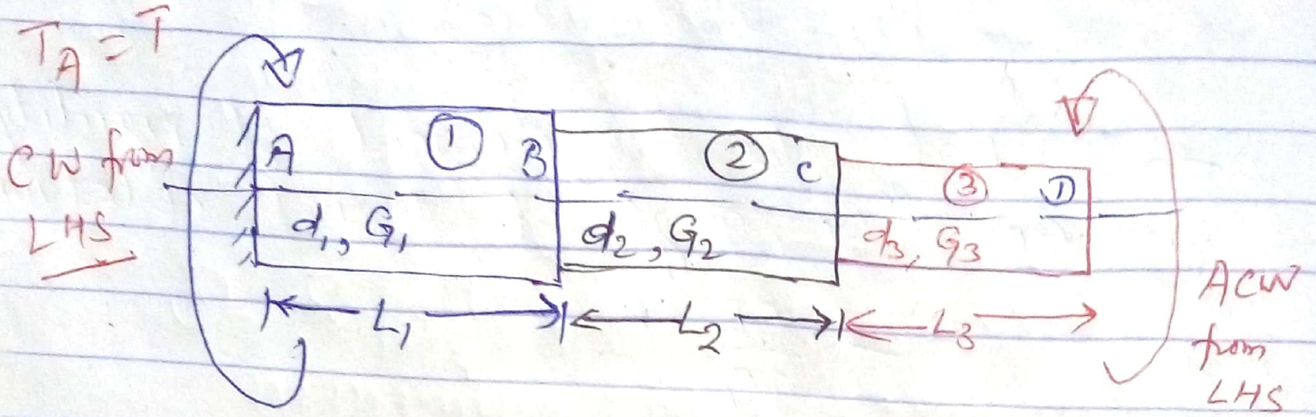
SHAFT IN SERIES



* Load applied at free end

* Torsion equation cannot be applied because of non-prismatic shaft

* Always θ is max^m at the free end

* $\theta_{fixed} = 0$

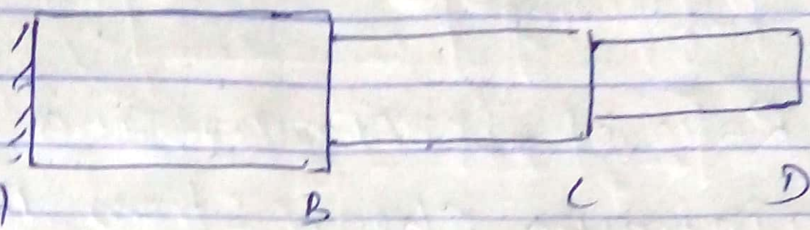
* $T_1 = T_2 = T_3 = T$

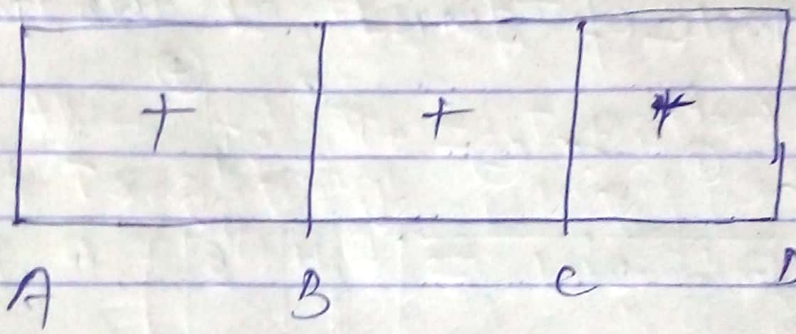
$$\theta_{total} = \theta_3 + \theta_2 + \theta_1$$

$$\theta_{DA} = \theta_{DC} + \theta_{CB} + \theta_{BA}$$

θ_{DA}: D → free end, A → fixed end

Torque diagram





① $\tau_{max} = \tau_3 = \dfrac{16 T_3}{\pi d_3^3} = \dfrac{16 T}{\pi d_3^3}$ — ①

② $\tau_{min} = \tau_1 = \dfrac{16 T_1}{\pi d_1^3} = \dfrac{16 T}{\pi d_1^3}$ — ②

$$\frac{\tau_{max}}{\tau_{min}} = \frac{\tau_3}{\tau_1} = \left(\frac{d_1}{d_3}\right)^3 =$$

$$= \left(\frac{d_{max}}{d_{min}}\right)^3$$

$$\theta_{total} = \theta_3 + \theta_2 + \theta_1$$

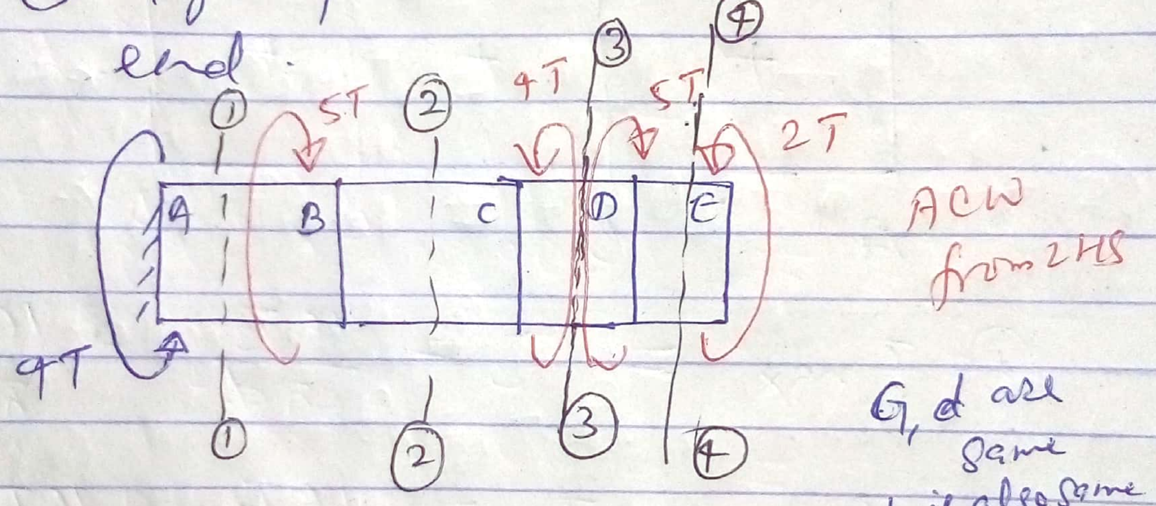
$$= \frac{T_3 L_3}{G_3 J_3} + \frac{T_2 L_2}{G_2 J_2} + \frac{T_1 L_1}{G_1 J_1}$$

$$\boxed{\theta_{total} = T\left(\sum_{i=1}^{3} \frac{L_i}{G_i J_i}\right)}$$

Q → For the compound shaft as shown in figure. Determine the following

(a) Ratio of maximum and minimum shear stress induced

(b) Angle of twist at the free end



Soln → By MOS (method of section)

CW = +ve

ACW = −ve

Step I :→ Reaction torque = ?

$T_A = -(\text{Net torque})$

$= -(-2T + 5T - 4T + 5T)$

$= -(4T)$

$\therefore \; T_A = 4T$ (ACW from LHS)

(i) $\tau_{max} = \dfrac{16 T_{max}}{\pi d^3}$

$= \dfrac{16 \times 4T}{\pi d^3}$. MPa

(ii) $\tau_{min} = \dfrac{16 T_{min}}{\pi d^3} = \dfrac{16 \times T}{\pi d^3}$

(iii) $\dfrac{\tau_{max}}{\tau_{min}} = \dfrac{T_{max}}{T_{min}} = \dfrac{4T}{T} = 4$

(iv) $\theta_{total} = \theta_4 + \theta_3 + \theta_2 + \theta_1$

$= \dfrac{L}{GJ}\left(T_1 + T_2 + T_3 + T_4 \right)$

$= \dfrac{L}{GJ}\left[-4T + T + (-3T) + 2T \right]$

$= \dfrac{1}{GJ}\left[-4T \right]$

$\therefore \theta_{total} = \dfrac{128\, TL}{\pi G d^4}$

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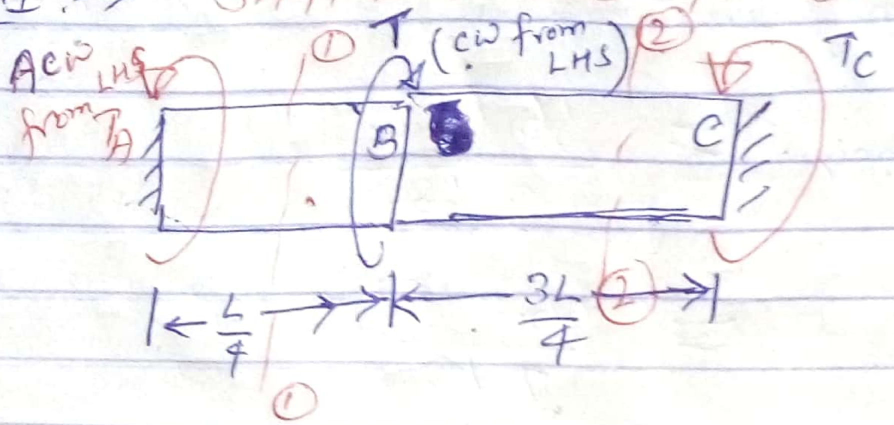
Case III :$\Rightarrow$ SHAFT FIXED AT BOTH END



G, J are same for both shaft

$*\quad T_1 = -T_A \;;\; T_2 = -T_A + T$

$*\quad T_A + T_C = T \quad —①$

$*\quad \theta_{total} = \theta_1 + \theta_2 = 0 \quad —②$

From eqn ②

$$\theta_1 = -\theta_2$$

$$\frac{T_1 L_1}{\cancel{G_1} J_1} = \frac{-T_2 L_2}{\cancel{G_2} J_2}$$

$$\Rightarrow -T_A \cdot \frac{L}{4} = -(-T_A + T)\frac{3L}{4}$$

$$\Rightarrow T_A L = \frac{3T \cdot L}{4}$$

$$\Rightarrow \boxed{T_A = \frac{3T}{4}}$$

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$$T_c = T - T_A$$
$$= 1 - \frac{3T}{4} = \frac{T}{4}$$
$$\boxed{T_c = \frac{T}{4}}$$

$$\therefore T_1 = -T_A = -\frac{3T}{4}$$

$$T_2 = -\frac{3T}{4} + T = \frac{T}{4}$$

Critical member is AB as $T_1 = \frac{3T}{4}$ & J & G are same

$$\boxed{\frac{\tau_{max}}{\tau_{min}} = \frac{\tau_1}{\tau_2} = \frac{T_1}{T_2} = 3}$$

Relative angular twist at B

$$= \frac{T_1 L_1}{GJ} = -\frac{3T}{4} \times \frac{L}{4} \times \frac{1}{GJ}$$

$$= -\frac{3TL}{16GJ}$$

(B) 2nd Method if $G_1 J_1 = G_2 J_2$

Only valid in this case

Reaction Torque at one fixed point

$$= \pm \frac{(\text{Applied torque}) \times (\text{Distance between applied torque and another fixed point})}{\text{Distance between fixed end}}$$

$$T_A = \frac{+T \cdot \frac{3L}{4}}{L} = \frac{3T}{4}$$

$$T_c = \frac{T \cdot \frac{L}{4}}{L} = \frac{T}{4}$$

$$\text{or } T_c = T - T_A = \frac{T}{4}$$

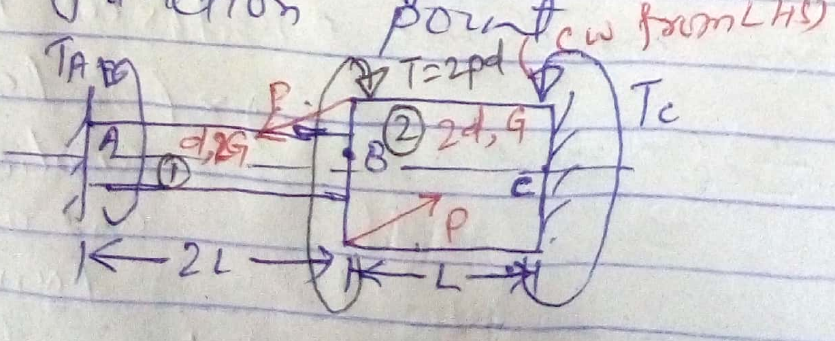
Q → For the ~~composite~~ shaft as shown in the figure, determine the following

(a) Reaction torque

(b) max^m shear stress ~~induced in the~~ shaft

(c) Ratio of the maximum and minimum shear stress

(d) Relative angular twist at the junction point



* $T_1 = -T_A$ $\boxed{T_A + T_C = T}$

* $T_2 = -T_A + T$

* $\theta_1 + \theta_2 = 0$

$\Rightarrow \boxed{\theta_1 = -\theta_2}$

$$\frac{T_1 L_1}{G_1 J_1} = -\frac{T_2 L_2}{G_2 J_2}$$

$$= \frac{\cancel{-}T_A \cancel{2}L}{\cancel{2}G \cdot \left(\frac{\pi}{32} d^4\right)} = \frac{\cancel{-}(-T_A + T) L}{G \cdot \left\{\frac{\pi}{32}(2d)^4\right\}}$$

$$\Rightarrow \frac{T_A \cancel{K}}{\cancel{d^4}} = \frac{(-T_A + T)\cancel{K}}{16 \cancel{d^4}}$$

$\Rightarrow 16 T_A = -T_A + T$

$\Rightarrow 17 T_A = T$

$\Rightarrow \boxed{T_A = \frac{T}{17}}$

$\therefore T_C = T - T_A = T - \frac{T}{17}$

$\boxed{T_C = \frac{16T}{17}}$

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$\therefore T_1 = -T_A = -\frac{T}{17}$

$T_2 = \frac{16T}{17}$

$\tau_1 = \frac{T_1}{Z_{P_1}} = \frac{-T/17}{\frac{\pi}{16} d^3} = \frac{-16T}{17\pi d^3}$

$\tau_2 = \frac{T_2}{Z_{P_2}} = \frac{\frac{16T}{17}}{\frac{\pi}{16}(2d)^3} = \frac{32T}{17\pi d^3}$

$\therefore \tau_{max} = \tau_2 = \frac{32T}{17\pi d^3}$

$$\boxed{\frac{\tau_{max}}{\tau_{min}} = \frac{\tau_2}{\tau_1} = 2}$$

Relative angular twist (B)

$\theta_1 = \frac{T_1 L_1}{G_1 J_1} = \frac{-\frac{T}{17} \times 2L}{2G \times \frac{\pi}{32} d^4}$

$= \frac{-32 \times (P \times 2d) L}{17\pi G d^4}$

$= \frac{-64 PL}{17\pi G d^3}$