More examples on numerical integration

Ex→2 Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\,dx$ using Simpson's $\frac{3}{8}$th rule

Soln:- $y = \sin x - \log_e x + e^x$

Let us divide the interval $[0.2, 1.4]$ into multiple of 3 into 6 equal parts.

→ multiple of 3

(4)

Thus

$$h = \frac{1.4 - 0.2}{6} = \frac{1.20}{6} = 0.2$$

No	x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
	y	3.0295 (y_0)	2.7975 (y_1)	2.8976 (y_2)	3.1660 (y_3)	3.5597 (y_4)	4.0698 (y_5)	4.4042 (y_6)

By Simpson's $\frac{3}{8}$ rule

$$\int_{0.2}^{1.4} y\,dx = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

(other than multiple of 3 → multiple of 3)

$$= \frac{3h}{8}\left[= 3\times\frac{0.2}{8}\left[(3.0295+4.4042) + 3(2.7975+2.8976+3.5597+4.0698) + 2\times 3.1660\right]\right.$$

$$= 4.053$$

EX→3 The velocity v of a particle at distance from a point on its linear path is given by the following table

(Sub interval)

s	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
V (cm/sec)	16 (y_0)	19 (y_1)	21 (y_2)	22 (y_3)	20 (y_4)	17 (y_5)	13 (y_6)	11 (y_7)	9 (y_8)

Estimate the time taken by the particle to traverse the distance of 20 meters by Simpson's rule.

Solⁿ:— Since number of sub interval is 8

Thus it can't be solved by Simpson's $\frac{3}{8}$ rule because 8 is not multiple of 3

Thus it may be solved by Trapezoidal rule or Simpson's $\frac{1}{3}$ rule.

Thus we know that Simpson's $\frac{1}{3}$ rule gives least error than Trapezoidal rule NOW by Simpson's $\frac{1}{3}$ rule

(8)

We know that $\frac{ds}{dt} = v \Rightarrow \frac{dt}{ds} = \frac{1}{v}$

then $\left[t\right]_{s=0}^{20} = \int_0^{20} \frac{1}{v}\, ds$

Here $h = 2.5$ and $n = 8$

Now by Simpson's $\frac{1}{3}$ rule

$$\int_0^{20} \frac{1}{v} ds = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{2.5}{3}\left[\left(\frac{1}{16} + \frac{1}{20}\right) + 4\left(\frac{1}{19} + \frac{1}{22} + \frac{1}{17} + \frac{1}{11}\right) + 2\left(\frac{1}{21} + \frac{1}{20} + \frac{1}{13}\right)\right]$$

= which gives required time.

Assignment

Ex→1 Calculate the value of $\int_0^{\pi} \sin x\, dx$ by Simpson's 1/3 rule, using 11 ordinates, verify your answer by direct integration

Ex→2 Given that

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

evaluate $\int_4^{5.2} \log x\, dx$ by (a) Trapezoidal rule (b) Simpson's $\frac{1}{3}$ rule (c) Simpson's 3/8 rule.

Ex→3 A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$ rule, find the velocity of the rocket at $t = 80$ seconds.

t (sec)	0	10	20	30	40	50	60	70
f (cm/sec^2)	30	31.63	33.34	35.47	37.75	40.33	43.25	46.65

80
50.67