

Hartley Oscillator (Radio frequency generator)

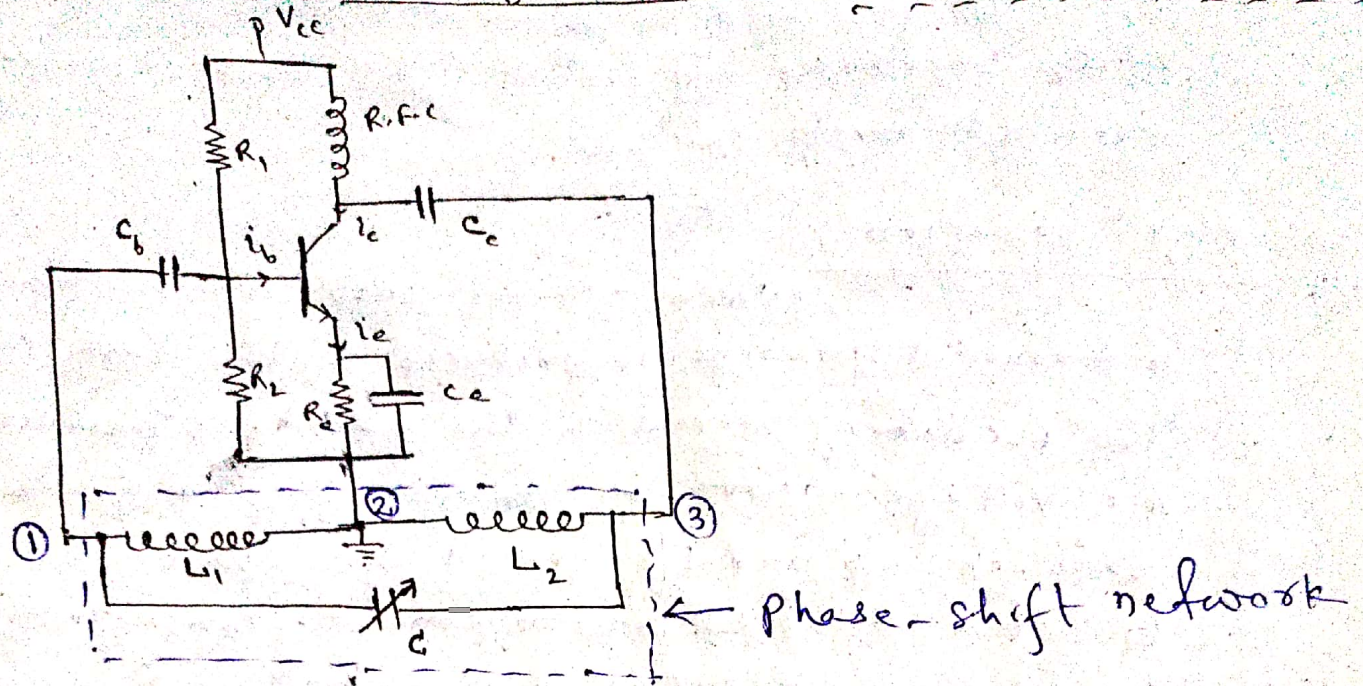


Fig-1.

Above fig-1 represents Hartley oscillator most commonly used as local oscillator in radio receivers. It can generate signal of frequency 10kHz to 100MHz.

The circuit consists of NPN-transistor with their biasing circuit elements R_1 , R_2 and R_e . The Radio frequency choke (R.F.C.) connected between supply and collector offers very high impedance to high frequency current and acts like short-circuit for d.c component. The frequency determining network is a parallel resonant circuit consists of L_1 , L_2 and a variable capacitor C . The junction of L_1 and L_2 is earthed. One side L_1 is connected to base via C_b . So, L_1 is in input side.

Similarly, one end of L_2 is connected to collector or C_c contributes into output side. These two parts are inductively coupled.

Working operation :-

When, collector supply is switched on, a transient current is produced in the tank circuit. The oscillatory current in the tank ckt produces a.c voltage across L_1 . In this way, a feedback between o/p and i/p circuit take place through inductive coupled arrangement. Consequently, there is a phase shift of 180° betⁿ o/p and i/p. The common-emitter amplifier also produces further 180° phase shift betⁿ i/p and o/p. Thus, total phase shift becomes $360^\circ (2\pi)$. This makes feedback positive which is the essential condition for oscillation. When, $|A\beta|$ is slightly greater than unity, oscillation developed in the circuit.

Frequency of oscillation (Mathematical analysis)

For, the analysis, we made following assumptions

i) h_{oe} of transistor is negligibly small, hence $h_{oe} V_o \approx 0$

ii) h_{oe} is very small and hence $\frac{1}{h_{oe}}$ is very large.

Now, circuit as in fig-1 can be equivalently redrawn as shown in fig-2.

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OR

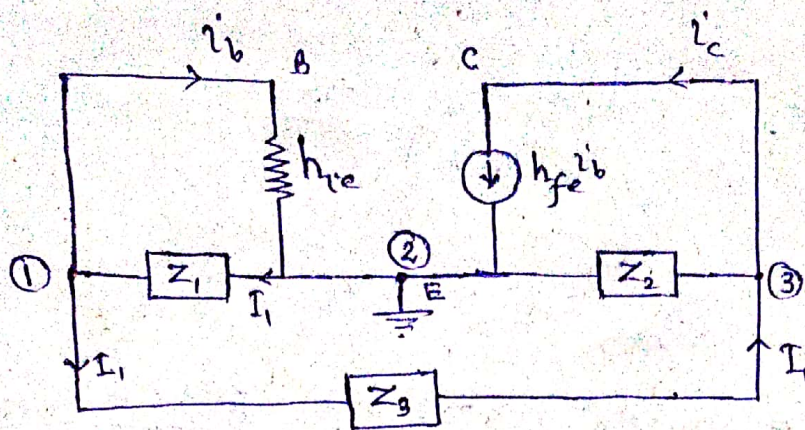


Fig-2

We, take load impedance Z_L betⁿ collector and emitter terminals. i.e betⁿ the terminals (2) and (3). It is clear, from fig-2 that, Z_1 and h_{ie} are parallel ($Z_1 || h_{ie}$). Their impedance ($Z'_1 = Z_1 || h_{ie}$) is in series with Z_3 . i.e ($Z'_1 + Z_3$) is in parallel with Z_2 .

Now,
$$Z'_1 = \frac{Z_1 h_{ie}}{Z_1 + h_{ie}} \quad \text{--- (I)}$$

or
$$Z'_1 + Z_3 = \left(\frac{Z_1 h_{ie}}{Z_1 + h_{ie}} \right) + Z_3 = \frac{Z_1 h_{ie} + Z_3 (Z_1 + h_{ie})}{Z_1 + h_{ie}}$$

or
$$Z'_1 + Z_3 = \frac{h_{ie} (Z_1 + Z_3) + Z_1 Z_3}{Z_1 + h_{ie}} \quad \text{--- (II)}$$

Now, load impedance Z_L is given by —

or
$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{(Z'_1 + Z_3)}$$

$$\text{or } \frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{\frac{h_{re}(Z_1+Z_3) + Z_1 Z_3}{Z_1 + h_{re}}}$$

$$\text{or } \frac{1}{Z_L} = \frac{1}{Z_2} + \frac{(Z_1 + h_{re})}{h_{re}(Z_1+Z_3) + Z_1 Z_3}$$

$$\text{or } \frac{1}{Z_L} = \frac{[h_{re}(Z_1+Z_3) + Z_1 Z_3] + Z_2(Z_1 + h_{re})}{Z_2 [h_{re}(Z_1+Z_3) + Z_1 Z_3]}$$

$$\text{or } \frac{1}{Z_L} = \frac{h_{re}(Z_1+Z_2+Z_3) + Z_1 Z_2 + Z_1 Z_3}{Z_2 [h_{re}(Z_1+Z_3) + Z_1 Z_3]}$$

$$\text{or } Z_L = \frac{Z_2 [h_{re}(Z_1+Z_3) + Z_1 Z_3]}{h_{re}(Z_1+Z_2+Z_3) + Z_1 Z_2 + Z_1 Z_3} \quad \text{--- (iii)}$$

Since we know that, from h-parameter approximate analysis —

$$A_v = - \frac{h_{fe} Z_L}{h_{re}} \quad \text{--- (iv); (see class notes)}$$

Now, feedback fraction β can be calculated as:

The output voltage betⁿ terminal (3) & (2); (fig-2)

$$V_o = (Z_1' + Z_3) I_1 = \left[\frac{h_{re}(Z_1+Z_3) + Z_1 Z_3}{Z_1 + h_{re}} \right] I_1$$

voltage feedback to the input terminals — (1) & (2)

$$V_{fb} = Z_1' I_1 = \left(\frac{Z_1 h_{re}}{Z_1 + h_{re}} \right) I_1$$

$$\therefore \beta = \frac{V_{fb}}{V_o} = \left\{ \left(\frac{Z_1 h_{re}}{Z_1 + h_{re}} \right) I_1 \right\} \div \left\{ \frac{h_{re}(Z_1+Z_3) + Z_1 Z_3}{Z_1 + h_{re}} \right\}$$

(A)

Or

$$\text{or } \beta = \frac{Z_1 h_{re}}{Z_1 Z_3 + h_{re}(Z_1 + Z_3)} \quad \text{--- (v)}$$

From Barkhausen criterion, frequency to be sustain, $A\beta = 1$

Putting value of A from eqⁿ (iv) and β from (v)

$$\text{or } - \frac{h_{fe} Z_L}{h_{re}} \cdot \frac{Z_1 h_{re}}{Z_1 Z_3 + h_{re}(Z_1 + Z_3)} = 1$$

$$\text{or } - \frac{h_{fe} Z_L Z_1}{Z_1 Z_3 + h_{re}(Z_1 + Z_3)} = 1$$

Substituting value of Z_L from eqⁿ (iii)

$$\text{or } \frac{-h_{fe} Z_1}{\{Z_1 Z_3 + h_{re}(Z_1 + Z_3)\}} \times \frac{Z_2 \{h_{re}(Z_1 + Z_3) + Z_1 Z_3\}}{h_{re}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3} = +1$$

$$\text{or } -h_{fe} Z_1 Z_2 = h_{re}(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_3$$

$$\text{or } h_{re}(Z_1 + Z_2 + Z_3) + Z_1 Z_3 + Z_1 Z_2 (1 + h_{fe}) = 0 \quad \text{--- (vi)}$$

Eqⁿ (vi) is called general eqⁿ of oscillation.

Here for Hartley oscillator,

$$Z_1 = j\omega L_1 + j\omega M = X_{L_1}$$

$$Z_2 = j\omega L_2 + j\omega M = X_{L_2}$$

$$Z_3 = \frac{1}{j\omega C} = X_C \Rightarrow Z_3 = -\frac{j}{\omega C}$$

; M = Mutual inductance of coil
; (see in fig-1.)

(5)
or

Substituting values of Z_1, Z_2 and Z_3 in eqⁿ (vi)

$$h_{ie} \left[j\omega L_1 + j\omega M + (j\omega L_2 + j\omega M) - \frac{j}{\omega C} \right] + (j\omega L_1 + j\omega M)(j\omega L_2 + j\omega M) \times \\ (1 + h_{fe}) + (j\omega L_1 + j\omega M) \cdot \left(-\frac{j}{\omega C} \right) = 0$$

$$\text{or } h_{ie} \left[j\omega(L_1 + M) + j\omega(L_2 + M) - \frac{j}{\omega^2 C} \right] + j\omega(L_1 + M) \times$$

$$j\omega(L_2 + M)(1 + h_{fe}) - j^2 \omega^2 (L_1 + M) \cdot \frac{1}{\omega^2 C} = 0$$

$$\text{or } j\omega h_{ie} \left[L_1 + M + L_2 + M - \frac{1}{\omega^2 C} \right] - \omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) \\ - \cancel{\omega^2} \times \left(\frac{L_1 + M}{C} \right) = 0$$

$$\text{or } j\omega h_{ie} \left[L_1 + L_2 + 2M - \frac{1}{\omega^2 C} \right] - \omega^2 (L_1 + M)(L_2 + M)(1 + h_{fe}) \\ + \left(\frac{L_1 + M}{C} \right) \times \frac{\omega^2}{\omega^2} = 0$$

$$\text{or } -\omega^2 (L_1 + M) \cdot \left\{ (L_2 + M)(1 + h_{fe}) - \frac{1}{\omega^2 C} \right\} +$$

$$j\omega h_{ie} \left[L_1 + L_2 + 2M - \frac{1}{\omega^2 C} \right] = 0 = 0 + j \cdot 0$$

Equating real part to real part, and imaginary part to imaginary part on both sides,

$$L_1 + L_2 + 2M - \frac{1}{\omega^2 C} = 0$$

$$\text{or } L_1 + L_2 + 2M = \frac{1}{\omega^2 C}$$

$$\text{or } \frac{1}{\omega^2 C} = (L_1 + L_2 + 2M) C$$

$$\text{or } \omega^2 = \frac{1}{(L_1 + L_2 + 2M) C}$$

$$\text{or } \omega = \frac{1}{\sqrt{(L_1 + L_2 + 2M) C}}$$

$$\Rightarrow \left[f = \frac{1}{2\pi \sqrt{(L_1 + L_2 + 2M) C}} \right]$$

i. Frequency of oscillation of Hartley oscillator is given by —

$$f = \frac{1}{2\pi \sqrt{(L_1 + L_2 + 2M)C}} \quad \text{--- (vii)}$$

Condition for maintenance of oscillation

Condition for maintenance of oscillation can be obtained from real part —

$$\therefore \omega^2 (L_1 + M) \left[(L_2 + M)(1 + h_{fe}) - \frac{1}{\omega^2 C} \right] = 0$$

$$\therefore \omega \neq 0, L_1 + M \neq 0$$

$$(1 + h_{fe})(L_2 + M) - \frac{1}{\omega^2 C} = 0 \quad \text{or } (1 + h_{fe}) = \frac{1}{\omega^2 C (L_2 + M)}$$

⑦

Ans

From previous discussion, we have

$$\frac{1}{\omega^2 C} = L_1 + L_2 + 2M.$$

$$\therefore (1 + h_{fe}) = \frac{1}{\omega^2 C (L_2 + M)} = \frac{(L_1 + L_2 + 2M)}{(L_2 + M)}$$

$$\text{or } h_{fe} = \frac{L_1 + L_2 + 2M}{L_2 + M} - 1$$

$$\text{or } \boxed{h_{fe} = \left(\frac{L_1 + M}{L_2 + M} \right)} \quad \text{--- (VIII)}$$

Eqⁿ (VIII) gives condition for maintenance of oscillation.

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