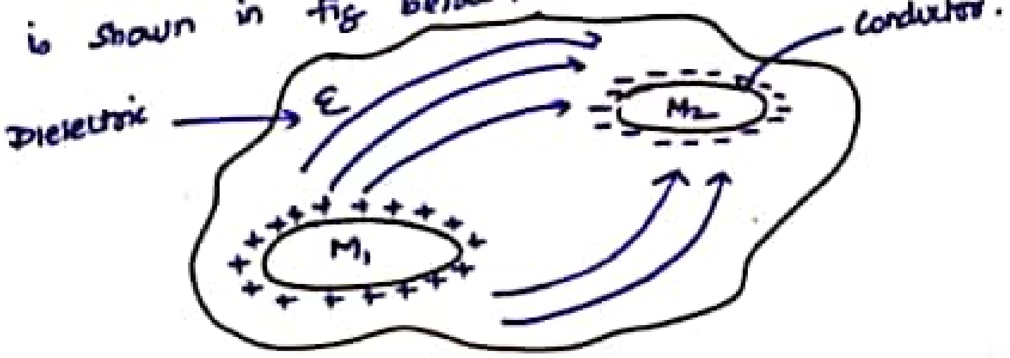
CONCEPT OF CAPACITANCE:-

Consider two conducting materials M_1 and M_2 which are placed in a dielectric medium having permittivity ε. The material M_1 carries a positive charge Q while the material M_2 carries a negative charge $-Q$ (equal in magnitude). There are no other charges present and the total charge of the system is zero.

In conductors, charge cannot reside within the conductor and it resides only on the surface. Thus for M_1 and M_2 charges $+Q$ and $-Q$ resides on the surfaces of M_1 and M_2 respectively. This is shown in fig below.



Such a system which has two conducting surfaces carrying equal and opposite charges seperated by a dielectric is called <u>capacitive systems</u> giving rise to a <u>capacitance</u>.

The electric field is normal to the conductor surface and the electric flux is directed from M_1 towards M_2 in such a system. There exist a potential difference b/w the two surfaces M_1 and M_2. Let this potential is V_{12}.

The ratio of magnitudes of the total charge on any one of the two conductors and potential difference b/w the conductors is called the capacitance. It is denoted by 'c'.

$$c = \frac{Q}{V_{12}}$$

$V_{12} = V_1 - V_2$
$V_2 = -ve$
$V_1 = +ve$

In general $c = \frac{Q}{V}$

where Q = charge in coulombs

V = Potential difference in volts.

The capacitance is measured in farads (F) and

$$1 \text{ Farad} = \frac{1 \text{ coulomb}}{1 \text{ Volt}}.$$

As charge Q resides only on the surface of the conductor it can be obtained from the Gauss's law as,

$$Q = \oint_S \vec{D}\cdot\vec{ds} = \oint_S \varepsilon_0\varepsilon_r\vec{E}\cdot\vec{ds} = \oint_S \varepsilon\vec{E}\cdot\vec{ds}$$

while V is the work done in moving unit positive charge from -ve to +ve surface and can be obtained as,

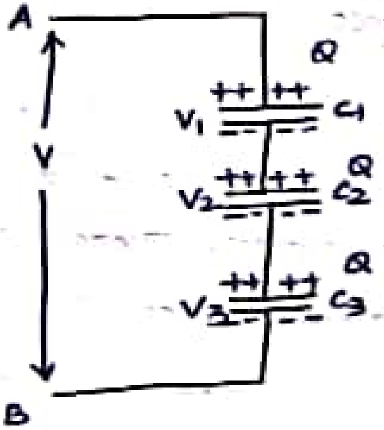
$$V = -\int_L \vec{E}\cdot\vec{dL} = -\int_-^+ \vec{E}\cdot\vec{dL}$$

Hence capacitance can be expressed as.

$$c = \frac{Q}{V} = \frac{\oint_S \varepsilon\vec{E}\cdot\vec{ds}}{-\int_-^+ \vec{E}\cdot\vec{dL}} \text{ F}.$$

CAPACITORS IN SERIES:

consider the three capacitors in series connected across the applied voltage V as shown in figure below.

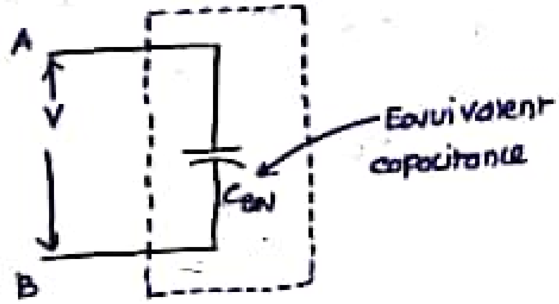


$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3.$$

Giving

$$V_1 = \frac{Q}{C_1} \; ; \; V_2 = \frac{Q}{C_2} \; ; \; V_3 = \frac{Q}{C_3}$$

If an equivalent capacitor also stores the same charge, when applied with the same voltage, then it is obvious that,



$$C_{eq} = \frac{Q}{V} \text{ (or)}$$

$$V = \frac{Q}{C_{eq}}$$

But $V = V_1 + V_2 + V_3$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

It is easy to find V_1, V_2 and V_3 if Q is known

for 'n' capacitors in series

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

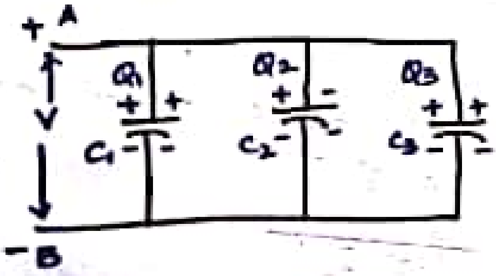
CAPACITORS IN PARALLEL:

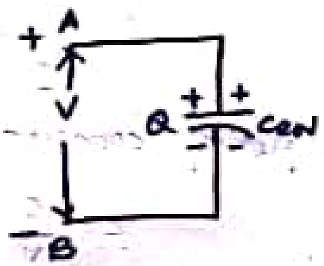
When capacitors are in $\parallel^{el}$, the same voltage exists across them, but charges are different.

$Q_1 = C_1 V$

$Q_2 = C_2 V$

$Q_3 = C_3 V$





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The Total charge stored by the Parallel bank of capacitors Q is given by

$$Q = Q_1 + Q_2 + Q_3$$
$$= C_1V + C_2V + C_3V$$
$$C_{eq}V = (C_1+C_2+C_3)V$$
$$\therefore \boxed{C_{eq} = C_1 + C_2 + C_3}$$

For n capacitors in $\parallel^{el}$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \dots + C_n}$$

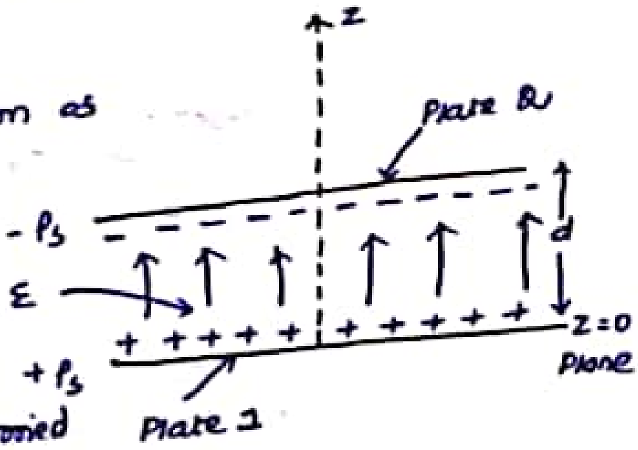
PARALLEL PLATE CAPACITORS:-

A Parallel Plate capacitor is shown in figure. It consists of two Parallel metallic plates seperated by distance 'd'. The space b/w the plates is filled with a dielectric of Permittivity ε.

The lower plate, plate 1 carries the positive charge and is distributed over it with a charge density $+\rho_s$. The upper plate, plate 2 carries the negative charge and is distributed over its surface with a charge density of $-\rho_s$. The plate 1 is Placed in z=0 i.e xy plane hence normal to it is z-direction. while upper plate 2 is in z=d Plane, Parallel to xy Plane.

Let A = Area of cross section of the Plates in m^2.

$$\boxed{\therefore Q = \rho_s A} \text{ coulombs.}$$



This is magnitude of charge carried on any one plate as charge carried by both is equal in magnitude.

To find Potential difference, let us obtain $\vec{E}$ b/w the Plates.

Assuming Plate 1 to be infinite sheet of charge

$$\vec{E}_1 = \frac{\rho_s}{2\varepsilon}\vec{a}_N \quad \text{[Page No 16 of unit-II]}.$$

$$= \frac{\rho_s}{2\varepsilon}\vec{a}_z \ \text{V/m}.$$

The $\vec{E}_1$ is normal at the boundary b/w conductor and dielectric without any tangential component.

While for plate 2, we can write

$$\vec{E}_2 = \frac{-\rho_s}{2\varepsilon}\vec{a}_N = -\frac{\rho_s}{2\varepsilon}(-\vec{a}_z) = \text{V/m}.$$

The direction of $\vec{E}_2$ is downwards i.e in $-\vec{a}_z$ direction.

∴ In b/w Plates

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\rho_s}{2\varepsilon}\vec{a}_z + \frac{\rho_s}{2\varepsilon}\vec{a}_z \Rightarrow \frac{\rho_s}{\varepsilon}\vec{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{\varepsilon}\vec{a}_z}$$

The Potential difference is given by.

$$V = -\int_{-}^{+} \vec{E}\cdot\vec{dL} = -\int_{\text{upper}}^{\text{lower}} \frac{\rho_s}{\varepsilon}\vec{a}_z\cdot\vec{dL}$$

Now $\vec{dL} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z$ in cartesian system,

$$V = -\int_{\text{upper}}^{\text{lower}} \frac{\rho_s}{\varepsilon}\vec{a}_z\cdot\left[dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z\right]$$

$$= -\int_{\text{upper}}^{\text{lower}} \frac{\rho_s}{\varepsilon}\,dz = -\frac{\rho_s}{\varepsilon}[z]_d^0 = -\frac{\rho_s}{\varepsilon}(-d) = \frac{\rho_s d}{\varepsilon}.$$

$$\boxed{V = \frac{\rho_s d}{\varepsilon}} \text{ Volts}$$

∴ The capacitance is the ratio of charge Q to voltage V.

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\varepsilon}} = \frac{\varepsilon A}{d} \cdot F$$

Thus is $\varepsilon = \varepsilon_0 \varepsilon_r$

$$\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}\ F$$

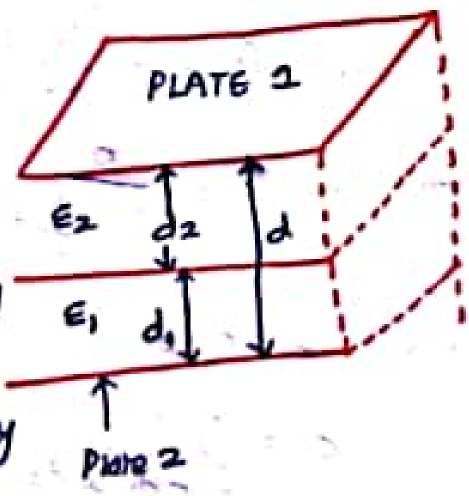
It can be seen that the value of capacitance depends on

(i) The permittivity of the dielectric used
(ii) The area of cross section of the plates
(iii) The distance of seperation of plates.

COMPOSITE PARALLEL PLATE CAPACITORS:

The composite parallel plate capacitor is one in which the space b/w the plates is filled with more than one dielectric.

Consider a composite capacitor with space filled with two seperate dielectrics for distance d_1 and d_2



The dielectric interface is parallel to the conducting plates.

The space d_1 is filled with dielectric having permittivity ε_1 while space d_2 is filled with dielectric having permittivity ε_2

Let Q = charge on each plate
$\vec{E}_1$ = Field intensity in region d_1
$\vec{E}_2$ = Field intensity in region d_2

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Both the intensities are uniform

$$\therefore V_1 = E_1 d_1$$
$$V_2 = E_2 d_2$$

where E_1 and E_2 are the magnitudes of the two intensities.

$$V = V_1 + V_2 = E_1 d_1 + E_2 d_2 \quad \cdots (1)$$

At a dielectric - dielectric interface, the normal components of flux densities are equal

i.e $D_{N1} = D_{N2}$

Now $D_1 = \varepsilon_1 E_1$ and $D_2 = \varepsilon_2 E_2$

$$\Rightarrow E_1 = \frac{D_1}{\varepsilon_1} \quad \& \quad E_2 = \frac{D_2}{\varepsilon_2} \quad \cdots (2)$$

sub (2) in (1) we get

$$V = \frac{D_1}{\varepsilon_1} d_1 + \frac{D_2}{\varepsilon_2} d_2 \quad \cdots (3)$$

The magnitude of surface charge is same on each plate hence

$$\rho_s = D_1 = D_2 \quad \cdots (4)$$

sub (4) in (3) we get

$$V = \frac{\rho_s}{\varepsilon_1} d_1 + \frac{\rho_s}{\varepsilon_2} d_2 \Rightarrow \rho_s \left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2}\right] \quad \cdots (5)$$

$$\cdots (6)$$

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