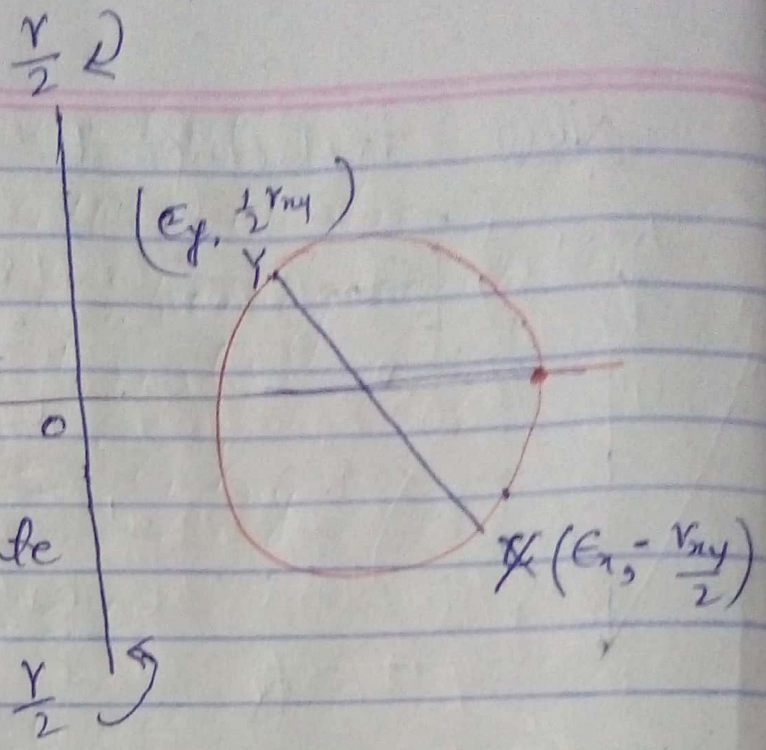
In the figure (A) as we assumed ϵ_x' is observed to rotate anticlockwise; while the side associated with ϵ_y is observed to rotate clockwise.



Thus if the shear causes a given side to rotate clockwise; the corresponding point on the mohr circle for plain is plotted above the horizontal axis and if the deformation causes the side to rotate counterclockwise the corresponding point is plotted below the horizontal axis.

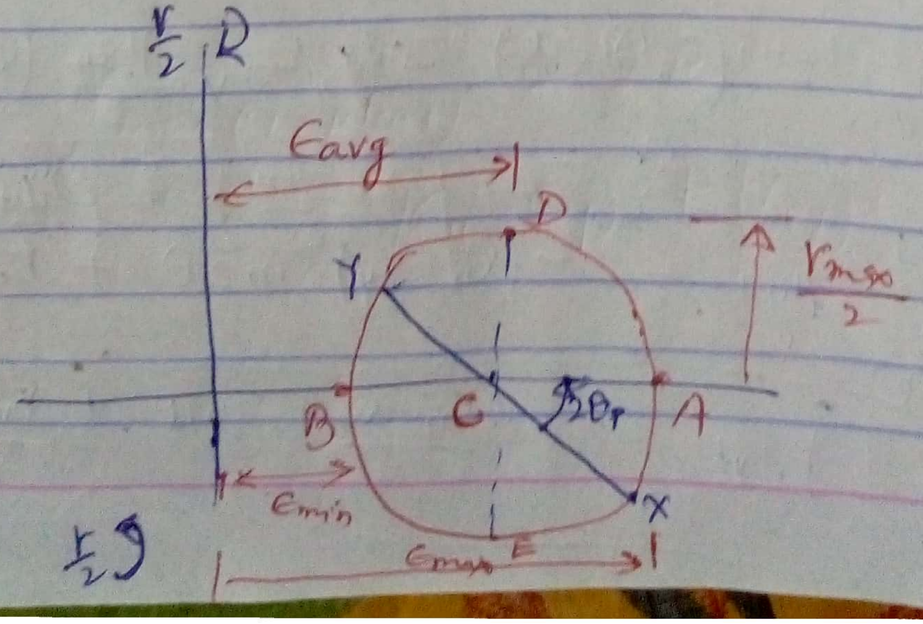


figure (D)

points A and B where Mohr's circle ~~intersect~~ intersects the horizontal axis corresponds to the principal strain ϵ_{max} and ϵ_{min}.

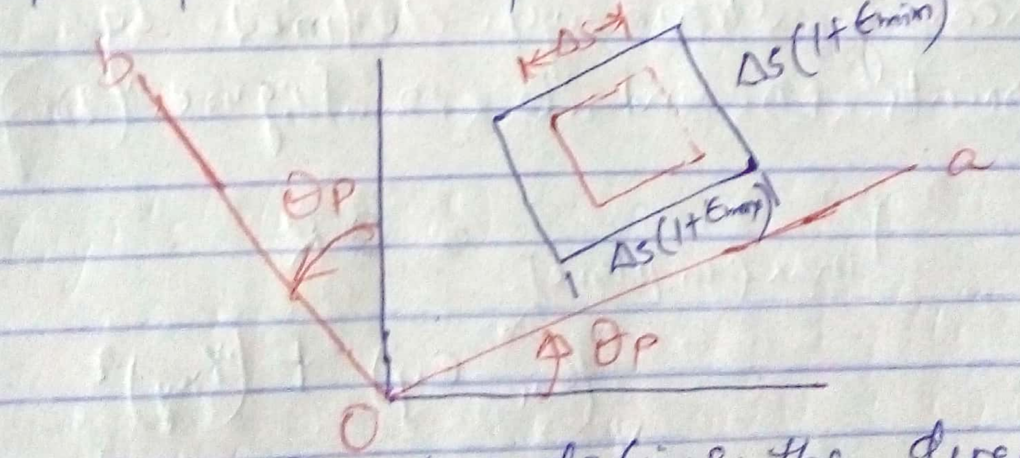
$$\epsilon_{max} = \epsilon_{avg} + R$$

$$\epsilon_{min} = \epsilon_{avg} - R$$

setting $\gamma_{x'y'} = 0$ then we have

$$\boxed{\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}}$$

The corresponding axes a and b in the following figure are the principal axes of strain



The angle θ_p which defines the direction of the principal axis Oa in the above figure the principal axis Oa corresponding to point A in figure (1) and equal to $\left(\frac{1}{2}\angle xCA\right)$

Also . we know that in the case of the elastic deformation of a homogeneous & isotropic materials,

$$\tau_{xy} = G \gamma_{xy}$$

Thus if $\gamma_{xy} = 0$

$\tau_{xy} = 0$

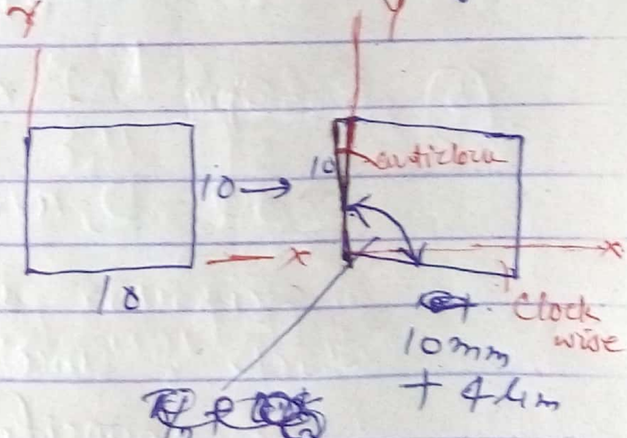
which indicate that the principal axes of strain coincide with the principal axes of stress.

The maximum in-plane shearing strain is defined by points D and E in figure (D)

$$\gamma_{max\,(in\,plane)} = 2R$$

$$= \sqrt{(\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2}$$

Q → In a material in a state of plane strain it is known that the horizontal side of a 10×10 mm square elongates by 4 μm while its vertical side remains unchanged and that at the lower left corner increases by 0.4×10^{-3} rad. Determine (a) the principal axes and principal strain (b) the maximum shearing strain and the corresponding normal strain.



Ans

Here $\epsilon_y = 0$

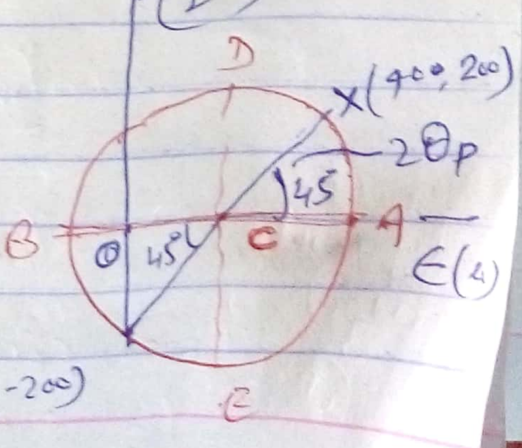
$$\epsilon_x = \frac{+4 \times 10^{-6}\ m}{10 \times 10^{-3}\ m}$$

$$= 400\ \mu$$

$$\left|\frac{\gamma_{xy}}{2}\right| = \frac{.4 \times 10^{-3}}{2}$$

$$= 2 \times 10^{-4} = 200\ \mu$$

$\left(\frac{\pi}{2} + 0.4 \times 10^{-3}\right)$ rad



$$OC = \frac{\epsilon_x + \epsilon_y}{2} = 200\ \mu$$

$OY = 200\ \mu$

$$\therefore R = \sqrt{(200)^2 + (200)^2}$$

$\simeq 283\ \mu$

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$$\epsilon_a = OA = OC + CA$$
$$= OC + R$$
$$= 200\mu + 283 = 483\mu$$

$$\epsilon_b = OB = OC - R$$
$$= 200 - 283 = -83\mu$$

since $OC = OY$

$$\therefore 2\theta_p = 45$$
$$\therefore \theta_p = 22.5^\circ$$

(b) Maximum shearing strain

point D and E defines the maximum in plane shearing strain which, since the principal strain have opposite signs, is also the actual maximum shearing strain.

$$\frac{\gamma_{max}}{2} = R = 283$$

$$\gamma_{max} = 2 \times 283 = 566\mu$$

The corresponding normal strains are both equal to $OC = 200\mu$