TRANSFORMATION OF PLANE STRAIN

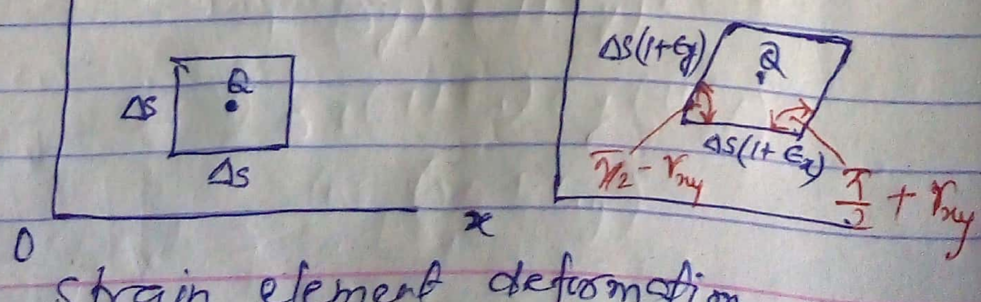
If the z-axis is chosen perpendicular to the planes in which the deformations takes place, we have

$$\epsilon_z = \gamma_{zx} = \gamma_{zy} = 0 \quad \text{and}$$

the only remaining strain components are ϵ_x, ϵ_y and γ_{xy}.

such situation is the case of plane strain.

Let us assume that a state of plane strain exists at point Q (with $\epsilon_z = \gamma_{zx} = \gamma_{zy} = 0$), and that it is defined by the strain component ϵ_x, ϵ_y and γ_{xy} associated with the x and y axis.

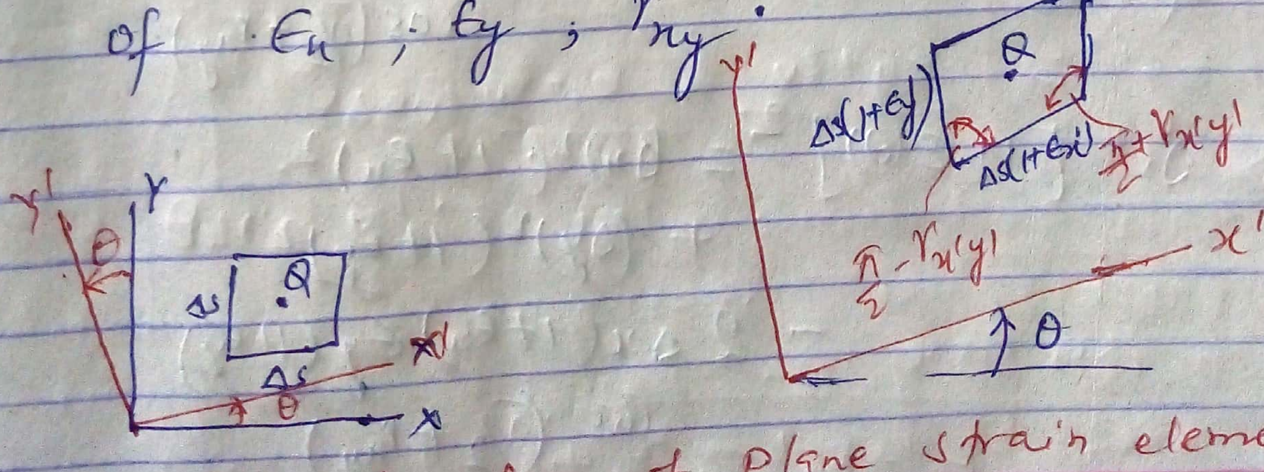


plane strain element deformation

Consider a square element of centre Q with sides of length Δs respectively $\parallel$r to the x and y-axis is deformed into a parallelogram with sides of length equal to $\Delta s(1+\epsilon_x)$ and $\Delta s(1+\epsilon_y)$ and forming angle of $(\pi/2 - \gamma_{xy})$ and $(\pi/2 + \gamma_{xy})$ respectively.

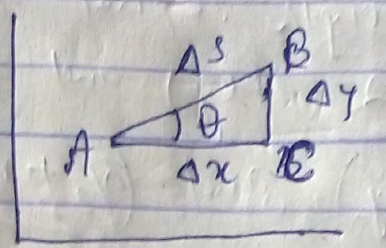
Now the frame of refrence is rotated through an angle θ.

Now our purpose is to determine $\epsilon_{x'}$, $\epsilon_{y'}$ and $\gamma_{x'y'}$ associated with the frame of refrence $x'y'$ obtained by rotating the x, & y axes through the angle θ in terms of ϵ_x ; ϵ_y ; γ_{xy}.

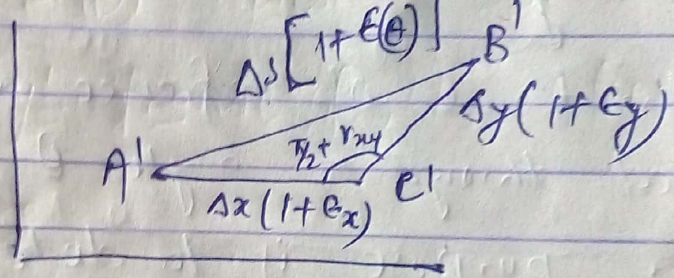


Transformation of plane strain element.

we first derive an expression for the normal strain $\epsilon(\theta)$ along a line AB forming an arbitrary angle θ with the x-axis



$\Delta x = \Delta s \cos\theta$
$\Delta y = \Delta s \sin\theta$



<u>Now</u> applying Cosine rule in $\Delta A'B'C'$

~~sot~~ we have

$$(A'B')^2 = (A'C')^2 + (B'C')^2 - 2(A'C')(B'C')\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)$$

$$\Rightarrow \Delta s^2[1+\epsilon(\theta)]^2 = (\Delta x)^2(1+\epsilon_x)^2 + (\Delta y)^2(1+\epsilon_y)^2 - 2\Delta x(1+\epsilon_x)\,\epsilon_y(1+\epsilon_y)\cos\left(\frac{\pi}{2}+\gamma_{xy}\right)$$

as; $\cos\left(\frac{\pi}{2} + \gamma_{xy}\right) = -\sin\gamma_{xy}$

Since γ_{xy} is very small, $\sin\gamma_{xy} = \gamma_{xy}$

$$\therefore \cos\left(\frac{\pi}{2} + \gamma_{xy}\right) \simeq -\gamma_{xy}$$

putting $\cos\left(\frac{\pi}{2} + \gamma_{xy}\right) \simeq -\gamma_{xy}$ in the above eqn, we have

$$(\Delta s)^2 \left[1 + \epsilon(\theta)\right]^2 = (\Delta s)^2 \cos^2\theta\,(1+\epsilon_x)^2 + (\Delta s)^2 \sin^2\theta\,(1+\epsilon_y)^2 - 2\cdot(\Delta s\cdot\cos\theta)(1+\epsilon_x)(\Delta s\sin\theta)\cdot(1+\epsilon_y)\cdot(-\gamma_{xy})$$

$$\Rightarrow \left[1 + \epsilon(\theta)\right]^2 = (1+\epsilon_x)^2\cos^2\theta + (1+\epsilon_y)^2\sin^2\theta + 2\sin\theta\cdot\cos\theta\cdot(1+\epsilon_x)(1+\epsilon_y)\cdot(\gamma_{xy})$$

Now second order term in $\epsilon(\theta)$, ϵ_x and ϵ_y and γ_{xy}

we have $1 + 2\epsilon(\theta) = (1 + 2\epsilon_x)\cos^2\theta + (1+2\epsilon_y)\sin^2\theta + (\sin 2\theta)(\gamma_{xy})(1)$

or $1 + 2\epsilon(\theta) = (\cos^2\theta + \sin^2\theta) + 2\epsilon_x\cos^2\theta + 2\epsilon_y\sin^2\theta + \gamma_{xy}\cdot\sin 2\theta$

or $2\epsilon(\theta) = 2\epsilon_x\cos^2\theta + 2\epsilon_y\sin^2\theta + \gamma_{xy}\sin 2\theta$

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Or, $\boxed{\epsilon(\theta) = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta}$

$$\boxed{\epsilon(\theta) = \frac{\epsilon_x + \epsilon_y}{2} \cos 2\theta + \frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cdot \sin 2\theta}$$

Since $\epsilon(\theta)$ represent $\epsilon_{x'}$ in the $x'y'$ frame of of refrence we have

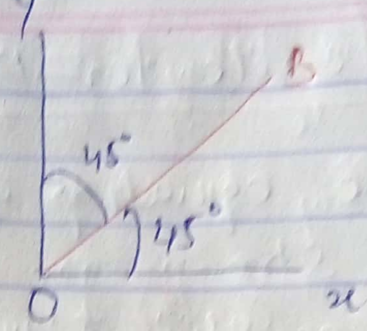
$$\boxed{\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta}$$

Similarly when θ is replaced by $\theta + 90°$ we obtain

$$\boxed{\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta}$$

In this case,

$$\boxed{\epsilon_x' + \epsilon_y' = \epsilon_x + \epsilon_y}$$



When $\theta = 0°$
we have

$$\epsilon(\theta) = \epsilon(0) = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos\theta + \frac{\gamma_{xy}}{2}\sin\theta$$

$$\therefore \epsilon(0) = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2} + 0$$

$$\boxed{\epsilon(0) = \epsilon_x}$$

When $\theta = 90°$

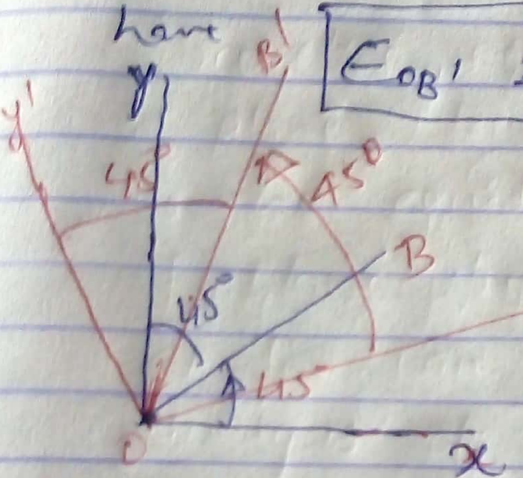
$$\epsilon(\theta) = \epsilon(90°) = \epsilon_y$$

$$\epsilon_{OB} = \epsilon(45°) = \frac{\epsilon_x+\epsilon_y}{2} + \cancel{\frac{\epsilon_{xy}}{2}} + \frac{\gamma_{xy}}{2}$$

$$\Rightarrow \boxed{\gamma_{xy} = 2\epsilon_{OB} - (\epsilon_x+\epsilon_y)}$$

This relation makes it possible to express the shearing strain associated with a given pair of rectangular axes in terms of the normal strain measured along these axes and their bisector.

Now replacing θ by $\theta + 45^\circ$ we have



$$\boxed{\epsilon_{OB'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta}$$

Now $\gamma_{x'y'} = 2\epsilon_{OB'} - (\epsilon_x' + \epsilon_y')$

$$\therefore \gamma_{x'y'} = 2\left(\frac{\epsilon_x + \epsilon_y}{2}\right) - 2\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + 2\cdot\frac{\gamma_{xy}}{2}\cos 2\theta - (\epsilon_x' + \epsilon_y')$$

$$\text{or } \gamma_{x'y'} = \cancel{(\epsilon_x' + \epsilon_y')} - \cancel{(\epsilon_x' + \epsilon_y')} - (\epsilon_x - \epsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\text{or } \boxed{\gamma_{x'y'} = -(\epsilon_x - \epsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta}$$

$$\text{or } \boxed{\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta}$$

Mohr's circle for plane strain

Since the equation for the transformation of plane strain are of the same form as the equation for the transformation of plane stress, the use of Mohr's circle can be extended to the analysis of plane strain.

we have

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \quad \text{— (A)}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \quad \text{— (B)}$$

$$\boxed{\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right) \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta} \quad \text{— (C)}$$

Let us suppose $\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2}$

and $R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$