CONTINUITY EQUATION:

The continuity equation of the current is based on the principle of conservation of charge.

The principle states that:

The charges can neither be created nor be destroyed.

Consider a closed surface S with a current density $\vec{J}$, then the total current I crossing the surface S is given by,

$$I = \oint_S \vec{J} \cdot \vec{ds} \quad \text{---} (1)$$

- The current flows outwards from the closed surface.
- The current means the flow of positive charges.
- The current I is constituted due to outward flow of +ve charge from the closed surface S.
- According to principle of conservation of charge, there must be decrease of an equal amount of +ve charge inside the closed surface.
- Hence the outward rate of flow of +ve charge gets balanced by the rate of decrease of charge inside the closed surface.

Let Q_i = charge within the closed surface

$-\frac{dQ_i}{dt}$ = Rate of decrease of charge inside the closed surface.

The negative sign indicates decrease in charge.

Due to principle of conservation of charge, this rate of decrease is same as rate of outward flow of charge, which is current

$$\therefore I = \oint_S \vec{J} \cdot \vec{ds} = -\frac{dQ_i}{dt} \quad \text{---} (2) \quad [\text{outward flowing current } I].$$

This is the integral ~~form~~ of the continuity equation of the current.

Current entering the volume is

$$\oint_S \vec{J} \cdot \vec{ds} = -I = \frac{dQ_i}{dt} \quad \text{---} (3)$$

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The point form of the continuity equation can be obtained from the integral form.

Using divergence theorem, convert the surface integral in integral form to the volume integral.

$$\left[\oint_S \vec{D}\cdot d\vec{s} = \int_{vol} (\nabla\cdot\vec{D})dv\right] \quad \therefore \oint_S \vec{J}\cdot d\vec{s} = \int_{vol} (\nabla\cdot\vec{J})dv \quad ---- (4)$$

$$\therefore \quad -\frac{dQ_i}{dt} = \int_{vol} (\nabla\cdot\vec{J})dv$$

But $Q_i = \int_{vol} \rho_v \, dv$ where $\rho_v \rightarrow$ volume charge density.

$$\therefore \int_{vol} (\nabla\cdot\vec{J})dv = -\frac{d}{dt}\left[\int_{vol} \rho_v \, dv\right] = -\int_{vol} \frac{d\rho_v}{\partial t} dv.$$

for constant surface derivative becomes partial derivative

$$\therefore \int_{vol} (\nabla\cdot\vec{J})dv = -\int_{vol} \frac{\partial \rho_v}{\partial t} dv. \quad --- (5)$$

If the above relation is true for any volume, it must be true for incremental volume Δv

$$\therefore (\nabla\cdot\vec{J})\Delta v = -\frac{\partial \rho_v}{\partial t}\Delta v$$

$$\therefore \boxed{(\nabla\cdot\vec{J}) = -\frac{\partial \rho_v}{\partial t}} \quad --- (6)$$

$\rightarrow$ Point form or differential form of continuity equation of the current.

For steady currents which are not the function of time $\frac{\partial \rho_v}{\partial t} = 0$ hence

$$(\nabla\cdot\vec{J}) = 0. \quad \text{for Steady State.}$$

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