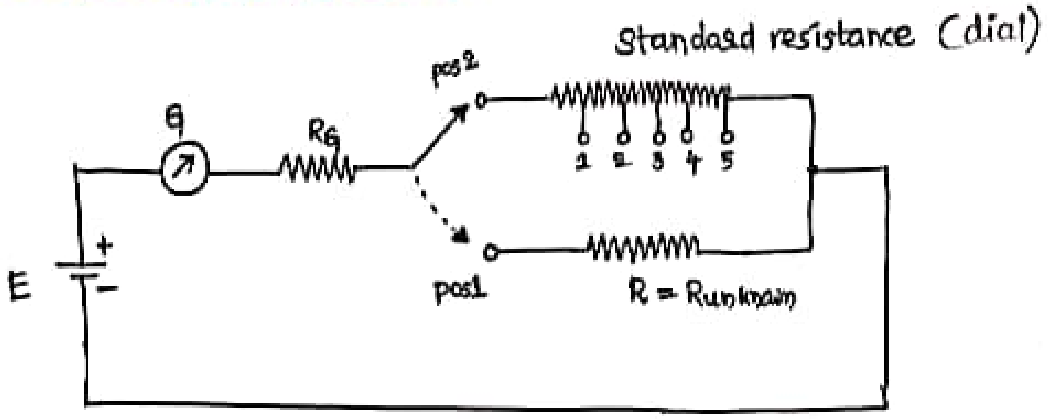
Substitution Method :-



case (i) :- switch is moved to position '1'. $\Rightarrow I_G = I_1$

$$\therefore \quad I_1 = \frac{E}{(R_G + R)} \quad — ①$$

case (ii) :- Switch is moved to position '2' $\Rightarrow I_G = I_2$

adjust the dial on standard resistance box...

ensure.. $I_2 = I_G = I_1$

$$\therefore \quad I_2 = \frac{E}{(R_G + r_{switch})} = I_1 = \frac{E}{(R_G + R)} \quad — ②$$

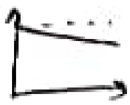
$\therefore$ Unknown resistance $(R) = r_{switch}$ (reading (or) dial resista

advantage :- One of the best method to measure resistance (pract

disadvantages :- * Choice of availability of standard resistance box

* For long time measurement in laboratories, standar

battery may be changed.



→ Let $\theta_1 \rightarrow$ deflection of (G) with unknown resistance

$$\theta_1 \propto \frac{1}{(R_G + R)} \rightarrow ③$$

Let $\theta_2 \rightarrow$ deflection of (G) with standard resistance.

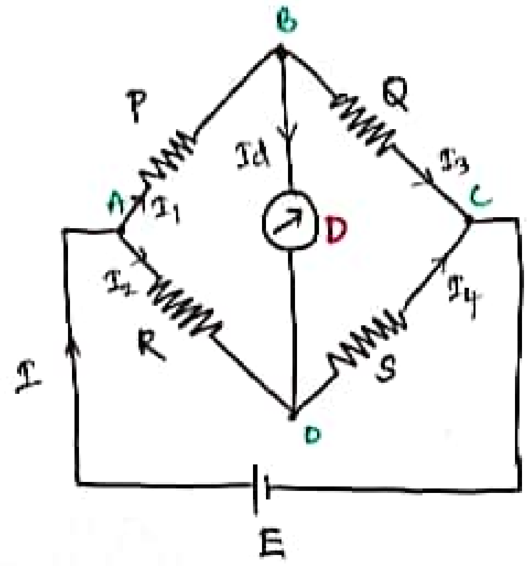
$$\frac{\theta_1}{\theta_2} = \frac{R_G + S}{R_G + R}$$

$$\therefore \boxed{R = \frac{\theta_2}{\theta_1}(R_G + S) - R_G}$$

E = constant (assumed).

Wheatstone Bridge :-

(Comparision method).



P, Q, S ⇒ known resistances

R ⇒ unknown resistance

E ⇒ DC battery.

P, Q, R, S ⇒ ratio arms resistance.

Ⓖ ⇒ Diagonal galvanometer detector.

$$e = V_{BD} = V_{BA} + V_{AD}$$

$$= V_{AD} - V_{AB}$$

$$e = I_2 R - I_1 P$$

$$\Rightarrow I_1 P = I_2 R$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{R}{P} \quad -①$$

(or) $e = V_{BD} = V_{BC} + V_{CD}$.

$$= V_{DC} - V_{BC}$$

$$e = S I_4 - I_3 Q \Rightarrow \frac{I_4}{I_3} = \frac{Q}{S} \quad -②$$

Unde bridge balance condition; $e = 0$., $I_d = 0$.

$$\therefore I_1 = I_3 \quad \& \quad I_2 = I_4$$

$$\therefore \frac{I_1}{I_2} = \frac{I_3}{I_4} \Rightarrow \therefore \text{from } ① \ \& ②$$

$$\frac{P}{R} = \frac{S}{Q} \Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}}$$

Detector sensitivity = voltage sensitivity = $\dfrac{\text{change in deflection}}{\text{change in voltage across detector}}$

$$\therefore S_V = S_D = \frac{\Delta\theta}{\Delta V} = \frac{\Delta\theta}{e} \Rightarrow \Delta\theta = e\,S_V$$

Bridge sensitivity = $\dfrac{(\text{change in deflection})}{(\text{unit change in resistance})} = \dfrac{\Delta\theta}{\left(\frac{\Delta R}{R}\right)} = S_B$.

$$\therefore \Delta\theta = S_B\left(\frac{\Delta R}{R}\right)$$

$$\therefore e\,S_V = S_B\left(\frac{\Delta R}{R}\right) \Rightarrow \boxed{S_B = \frac{R\,e\,S_V}{\Delta R}} \;**$$

at unbalanced point $(R' = R + \Delta R)$

$$e = V_{th} = V_{oc} = V_{AD} - V_{AB} = V_B - V_D$$

$$\boxed{e = E\left(\frac{R'}{R'+S} - \frac{P}{P+Q}\right)} \;** \quad \text{(or)} \quad e = E\left(\frac{R'}{R'+S} - \frac{P}{P+Q}\right)$$

$R' = R + \Delta R$.

$$e = E\left(\frac{(R+\Delta R)}{(R+S+\Delta R)} - \frac{P}{P+Q}\right)$$

$$\therefore e = E\left(\frac{R+\Delta R}{(R+S)+\Delta R} - \frac{R}{R+S}\right)$$

$$= E\left[\frac{(R+\Delta R)(R+S) - R[(R+S)+\Delta R]}{(R+S)^2 + \Delta R(R+S)}\right]$$

$$= E\left[\frac{\cancel{R^2} + \cancel{RS} + \cancel{\Delta R\cdot R} + \Delta R\cdot S - \cancel{SR} - \cancel{R^2} - \cancel{R\,\Delta R}}{(R+S)[(R+S)+\Delta R]}\right] = \frac{E(\Delta R\cdot S)}{(R+S)^2 + \Delta R R \ldots}$$

at balance point

$$\frac{P}{Q} = \frac{R}{S}$$

$$\frac{Q}{P} = \frac{S}{R}$$

$$\frac{Q}{P} + 1 = \frac{S}{R} + 1$$

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$e = \frac{S \cdot \Delta R \cdot E}{(R+S)^2 + \Delta R(R+S)}$$

$$(R+S)^2 >>> \Delta R(R+S)$$

$$\therefore \quad e = \frac{S \cdot \Delta R \cdot E}{R^2 + S^2 + 2SR} = \frac{(\Delta R)\, E}{\frac{R}{RS}\left(R^2 + S^2 + 2RS\right)}$$

$$e = \frac{R\, \Delta R \cdot E}{R\left(\frac{R^2}{S} + \frac{S}{R} + 2\right)}$$

$$\Delta V = e = \frac{\left(\frac{\Delta R}{R}\right) E}{\frac{R}{S} + 2 + \frac{S}{R}} \quad \text{(or)} \quad e = \frac{\left(\frac{\Delta R}{R}\right) E}{\frac{P}{Q} + 2 + \frac{Q}{P}}$$

$$S_V = \frac{\Delta \theta}{\Delta V} \; ; \Rightarrow \; S_V = \frac{\Delta \theta}{e} \; ; \quad S_B = \frac{\Delta \theta}{\left(\frac{\Delta R}{R}\right)}$$

$$\therefore \quad e\, S_V = \left(\frac{\Delta R}{R}\right) S_B$$

$$\frac{\left(\frac{\Delta R}{R}\right) E \cdot S_V}{\frac{P}{Q} + 2 + \frac{Q}{R}} = \left(\frac{\Delta R}{R}\right) S_B$$

$$\therefore \quad \boxed{S_B = \frac{e\, S_V}{\left(\frac{\Delta R}{R}\right)} = \frac{E \cdot S_V}{\frac{P}{Q} + 2 + \frac{Q}{R}}}$$

$S_{B\,max}$; if $\frac{P}{Q} = \frac{Q}{P} = 1$ i.e. $R = S = P = Q$

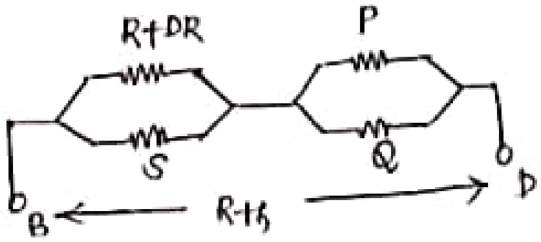
$$\therefore \boxed{S_{Bridge} = \frac{S_V E}{4}}$$

condition for obtaining max. bridge sensity, it is possible if the ratio of ratio arm resistances of ratio of arm (or) equal to unity. i.e. $P = Q = R = S$, the value is $\frac{E}{4}$ times that of detector sensitivity.

$$e = V_{th} = V_{oc} = \frac{E\left(\frac{\Delta R}{R}\right)}{\frac{P}{Q} + 2 + \frac{Q}{P}} = \frac{E\left(\frac{\Delta R}{R}\right)}{\frac{R}{S} + 2 + \frac{S}{R}} \quad \text{(or)}$$

$$\theta = E\left(\frac{R+\Delta R}{R+\Delta R+S} - \frac{P}{P+Q}\right).$$

$$R_{th} = S \parallel (R+\Delta R) + P \parallel Q$$

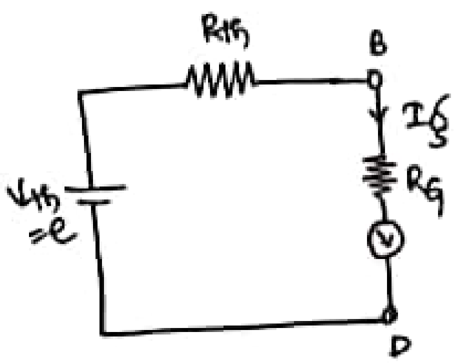


$$R_{th} = \frac{PQ}{P+Q} + \frac{S(\Delta R + R)}{R+S+\Delta R}$$

if $\Delta R = 0$

$$R_{th} = \frac{PQ}{(P+Q)} + \frac{RS}{(R+S)}$$

Thevenins equivalent ckt of Wheatstone Bridge



$$R_{th} = \frac{PQ}{(P+Q)} + \frac{RS}{(R+S)}$$

$$V_{th} = e = \frac{\left(\frac{\Delta R}{R}\right)E}{\frac{R}{S} + 2 + \frac{S}{R}} =$$

$$I_{th} = I_g = \frac{V_{th}}{R_{th} + R_g}$$

For ideal (G) ; $R_g = 0$ $\quad I_{th} = I_g = \frac{V_{th}}{R_{th}}$

→ If all arms are wheatstone Bridges of equal resistances then

$$R = P = Q = S.$$

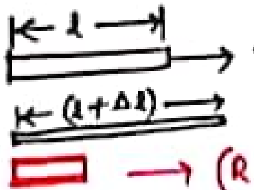
$$R_{th} = P \parallel Q + R \parallel S = R \parallel R + R \parallel R = \frac{R}{2} + \frac{R}{2} = R.$$

$$\therefore R_{th} = R, \quad V_{th} = V_{OC} = 0.$$

$$S_{B_{max}} = \frac{S_V E}{4}$$

→ $P = Q = S = R$; but one of the arm is under stress (strain).

case(i) :-



; length ↑ ; area ↓ but volume = constant.

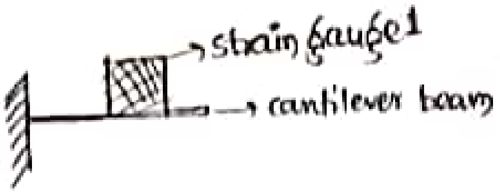
(compressive force)

$$R_{th} = \frac{(R+\Delta R)S}{(R+S+\Delta R)} + \frac{PQ}{P+Q} \quad ; \quad V_{th} = E\left(\frac{(R+\Delta R)}{(R+R+\Delta R)} - \frac{R}{R+R}\right)$$

$$= \frac{(R+\Delta R)R}{(R+R+\Delta R)} + \frac{R \cdot R}{R+R} \qquad = E\left(\frac{R^2 + R\Delta R}{2R+\Delta R} - \frac{1}{2}\right)$$

$$R_{th} = \frac{(R+\Delta R)R}{(2R+\Delta R)} + \frac{R}{2} \qquad = E\left[\frac{2R^2 + 2R\Delta R - 2R - \Delta R}{2(2R+\Delta R)}\right]$$

$$= E\frac{\Delta R}{2(2R+\Delta R)}$$



$$V_{th}' = E\left(\frac{\Delta R}{4R + 2\Delta R}\right)$$

$$4R >>> 2\Delta R.$$

$$\therefore V_{th} = \frac{E}{4}\frac{\Delta R}{R}$$

(Quarter bridge) equation ⟸ $\therefore \boxed{V_{th} = \frac{E}{4}\left(\frac{\Delta R}{R}\right)}$

$R = P = Q = S$; one beam $(R+\Delta R)$.

$$\boxed{S_Q = \frac{\Delta V}{\left(\frac{\Delta R}{R}\right)} = \frac{V_{th}}{\frac{\Delta R}{R}} = \frac{E}{4}}$$