

CURRENT:

- Flow of charges constitutes an Electric current
 - It can be measured by measuring how many charges are passing thro' a specified surface or a point in a material per second.
 - It is rate of flow of charge at a specified point or across a specified surface ^{per unit time} is called an Electric current.
 - It is measured in Ampere, which is Coulombs/sec (C/s).
- i.e. $I = \frac{dQ}{dt} \text{ C/s i.e Amps}$

A current of 1 Amp is said to be flowing across the surface when a charge of one Coulomb is passing across the surface in one second.

CURRENT DENSITY:

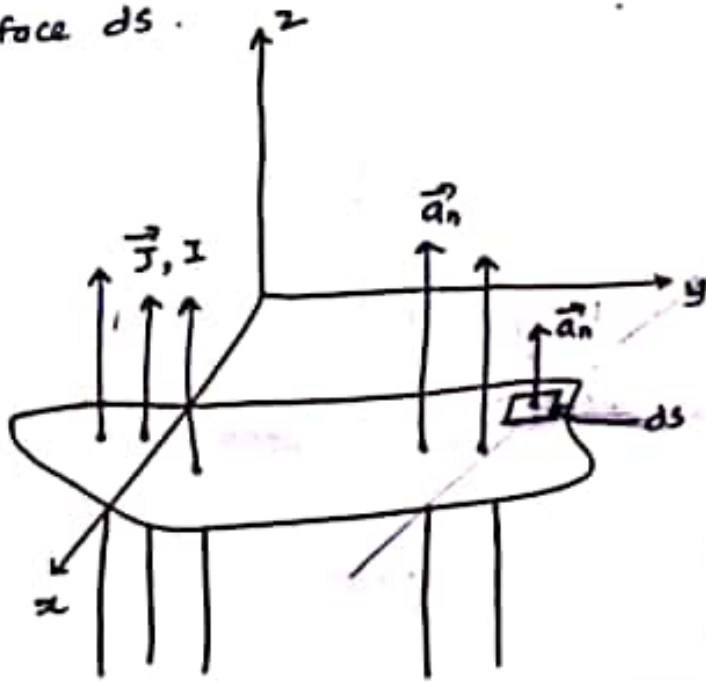
It is defined as the current passing thro' the unit surface area, when the surface is held normal to direction of the current.

- It is a vector quantity and denoted as $\vec{J}$.
- It is measured in Amperes per sq. meters (A/m^2).

RELATIONSHIP BETWEEN I AND J:

consider a surface S and I is the current passing thro' the surface. The direction of current is normal to the surface S and hence direction of $\vec{J}$ is also normal to the surface S .

consider an incremental area ds as shown in fig below and $\vec{a}_n$ is the unit vector normal to the incremental surface ds .



$$I = \int \vec{J} \cdot d\vec{s}$$

$$dI = \vec{J} \cdot d\vec{s}$$

$$d\vec{s} = ds \vec{a}_n$$

$$\vec{J} = J \vec{a}_n$$

Then the differential current dI passing thro' the differential surface ds is given by the dot product of the current density vector $\vec{J}$ and $d\vec{s}$.

$$\therefore dI = \vec{J} \cdot d\vec{s} \text{ [dot product]}$$

When $\vec{J}$ and $d\vec{s}$ are at right angles ($\theta = 90^\circ$) then

$$dI = \vec{J} \cdot d\vec{s} = |\vec{J}| |d\vec{s}| \cos 90^\circ$$

$$\boxed{dI = J ds}$$

and $I = \oint_S \vec{J} \cdot d\vec{s}$. $J \rightarrow$ current density in A/m^2

But if $\vec{J}$ is not normal to $d\vec{s}$ then the total current is obtained by integrating $\vec{J} \cdot d\vec{s}$

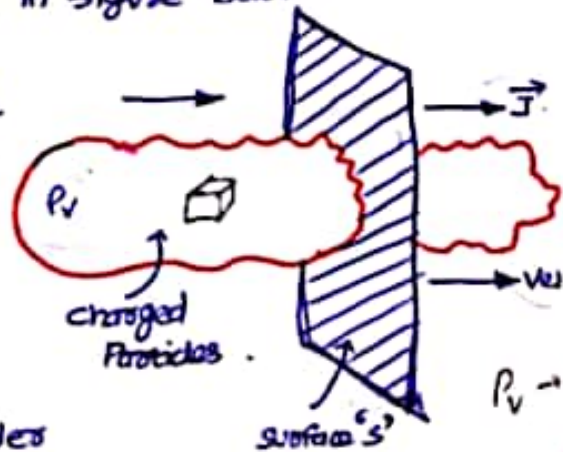
$$\boxed{I = \oint_S \vec{J} \cdot d\vec{s}}$$

RELATION BETWEEN $\vec{J}$ & P_V

(2)

The set of charged particles give rise to a charge density P_V in a volume V . The current density $\vec{J}$ can be related to the velocity with the volume charge density. i.e. charged particles in volume V crosses the surface S at a point. This is shown in figure below

The velocity with which the charge is getting transferred is $\vec{v}$ m/s. This is a vector quantity.



To derive the relation between $\vec{J}$ and P_V , consider differential volume ΔV having charge density P_V as shown in figure below. The elementary charge that volume carries is,

$$\Delta Q = P_V \Delta V \quad \text{--- (1)}$$

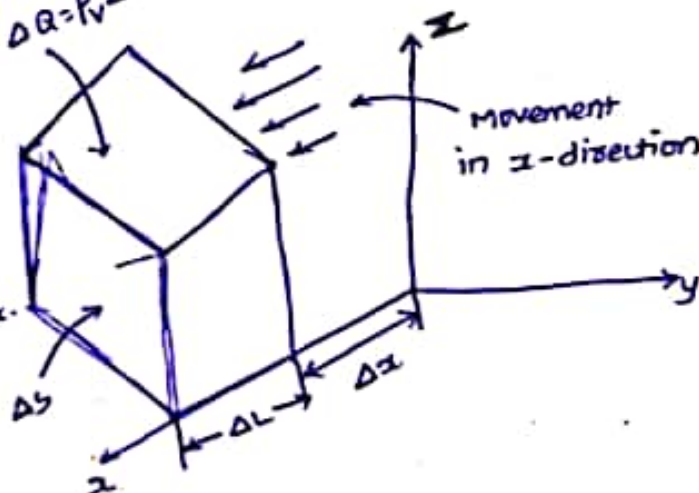
Let ΔL is the incremental length while ΔS is the incremental surface area & hence incremental volume is,

$$\Delta V = \Delta S \Delta L \quad \text{--- (2)}$$

$\therefore$ sub (2) in (1) we get

$$\Delta Q = P_V \Delta S \Delta L \quad \text{--- (3)}$$

Let the charge is moving in x -direction with velocity $\vec{v}$ and thus velocity has only x component v_x .



In the time interval Δt the element of charge has moved through distance Δx , in x direction. The direction is normal to the surface ΔS and hence the resultant current can be expressed as,

$$\Delta I = \frac{\Delta Q}{\Delta t} \text{ --- (4)}$$

But now, $\Delta Q = \rho_v \Delta S \Delta x$ as the charge corresponding the length Δx is moved and responsible for the current.

$$\therefore \Delta I = \rho_v \Delta S \frac{\Delta x}{\Delta t} \text{ --- (5)}$$

But $\frac{\Delta x}{\Delta t} = \text{Velocity in } x\text{-direction i.e. } v_x$

$$\therefore \Delta I = \rho_v \Delta S v_x \text{ --- (6)}$$

$\rightarrow$ x component of velocity $\vec{v}$

But $\Delta I = \vec{J} \Delta S$ when $\vec{J}$ and ΔS are normal.

Here $\vec{J}$ and ΔS are normal to each other hence comparing (7) and (6) we get

$$J_x = \rho_v v_x = x \text{ component of } \vec{J} \text{ --- (8)}$$

In general, the relation ship between $\vec{J}$ and ρ_v be expressed as

$$\vec{J} = \rho_v \vec{v}$$

where

$\vec{v}$ is the velocity vector.

