# * Fluid Mechanics - Module 04

# * Venturi meters

~~* Application of Bernoulli's equation~~

1. Venturimeter
2. orifice
3. pitot tube

(1) Venturimeter →

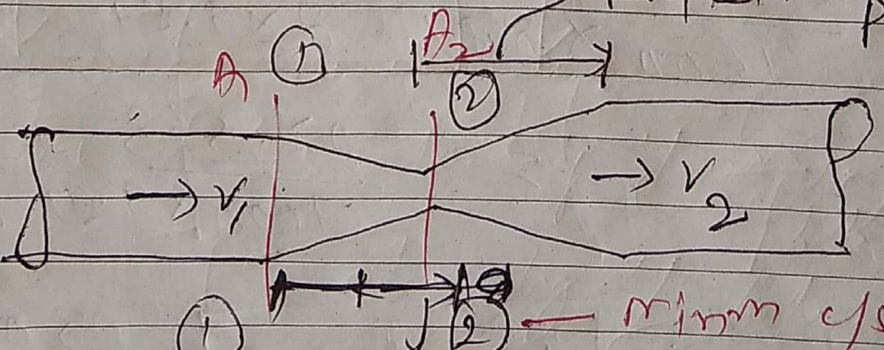
→ It is used measure flow Rate (discharge)

→ venturimeter is a short pipe consisting of two conical parts with a short portion of minm cross-sectional area.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

gradual increase or ~~Rapid~~ diverging portion.




Rapid converging portion

→ To reduce the loss of energy.

* In converging portion

(i) velocity increase along the converging

$A_1 V_1 = A_2 V_2$

$A_2 < A_1$

$$\boxed{V_2 = \frac{A_1}{A_2} V_1}$$

$\therefore \boxed{V_2 > V_1}$

(ii) the pressure decreases along the converging section

from Bernoulli's equation

$$\boxed{P_1 > P_2}$$

* for a horizontal venturimeter

$z_1 = z_2$

for Ideal fluid $\boxed{h_L = 0}$

$$\therefore \boxed{\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}}$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$$

$$\underline{v_2 > v_1}$$

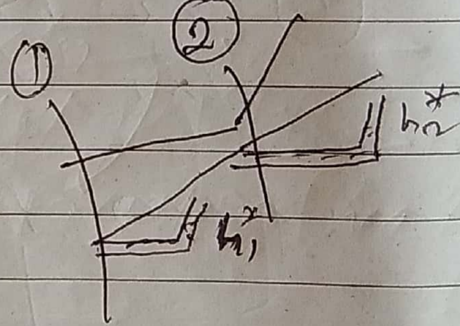
$$\frac{v_2^2}{2g} > \frac{v_1^2}{2g}$$

$$\underline{R.H.S > 0}$$

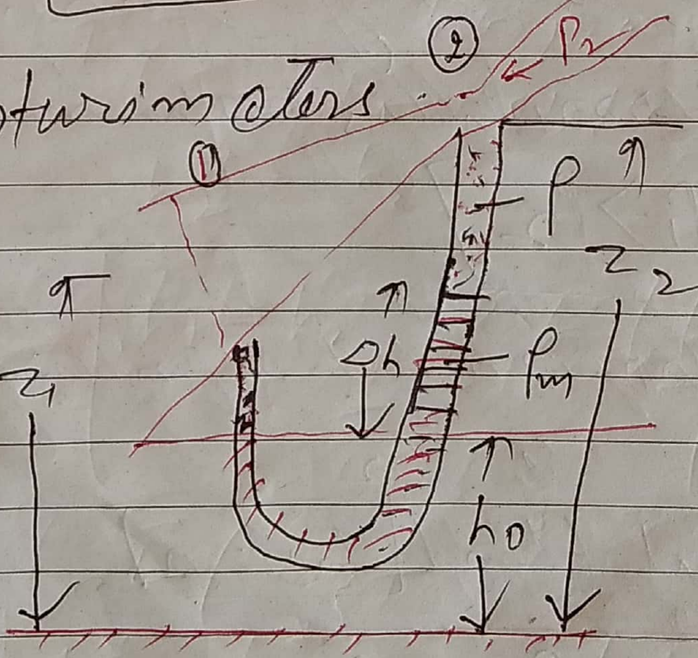
$$\rightarrow \underline{\frac{P_1}{\rho g} - \frac{P_2}{\rho g} > 0}$$

$$\rightarrow \frac{P_1}{\cancel{\rho g}} > \frac{P_2}{\cancel{\rho g}}$$

$$\therefore \boxed{P_1 > P_2}$$




* Venturimeters




where $\rho_m$ = density of manometric fluid

$\rho$ = density of fluid in pipe. or working fluid

* Applying Bernoullis equation



$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

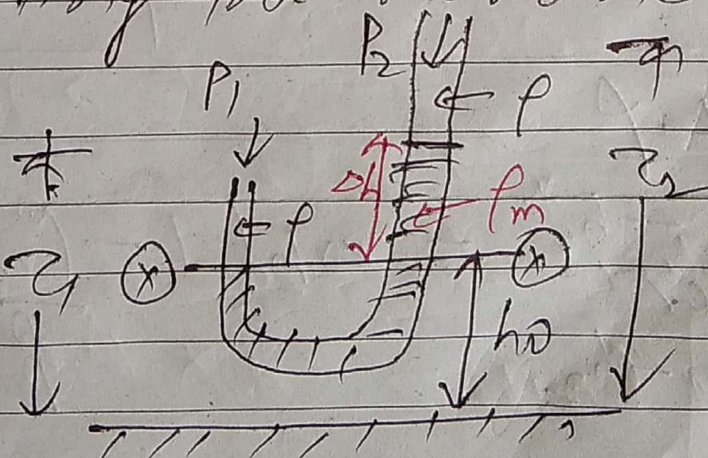
or $$\frac{V_1^2-V_2^2}{2g}=\frac{P_2}{\rho g}-\frac{P_1}{\rho g}+(z_2-z_1)$$

$$\therefore \frac{V_1^2-V_2^2}{2g}=\frac{P_2}{\rho g}+z_2-\left(\frac{P_1}{\rho g}+z_1\right)$$

or $$\frac{V_2^2-V_1^2}{2g}=\underbrace{\left(\frac{P_1}{\rho g}+z_1\right)}_{\text{piezometric head }(h^*)}-\left(\frac{P_2}{\rho g}+z_2\right)$$

$$\Rightarrow \boxed{\frac{V_2^2-V_1^2}{2g}=h_1^*-h_2^*} \quad (1)$$

* Writing the Manometer equation




$$P_1+\rho g(z_1-h_0)=P_2+\rho g(z_2-h_0-\Delta h)+\rho_m g\times\Delta h$$

$\rightarrow\ P_1 + \rho g z_1 - \rho g h_0 = P_2 + \rho g z_2 - \rho g h_0 - \rho g \Delta h + \rho_m g \Delta h$

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$\rightarrow\ \left(\frac{P_1}{\rho g} + z_1\right) - \cancel{h_0} = \frac{P_2}{\rho g} + z_2 - \cancel{h_0} - \Delta h + \frac{\rho_m}{\rho}\Delta h$

$\rightarrow\ h_1^* = h_2^* + \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$

$\therefore\ \boxed{h_1^* - h_2^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)}$

* If $z_1 = z_2$ (venturimeter is horizontal)

$h_1^* = h_1$ static pressure head.

$h_2^* = h_2$

$\rightarrow\ \boxed{h_1 - h_2 = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)}$

$\rightarrow\ \rho_m < \rho$

$h_1^* - h_2^* = \Delta h\left(1 - \frac{\rho_m}{\rho}\right)$
or
$h_1 - h_2$

from equation (1)

$$\frac{v_2^2 - v_1^2}{2g} = h_1^* - h_2^*$$

$$h_1^* - h_2^* \rightarrow \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\rightarrow \frac{v_2^2 - v_1^2}{2g} = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$$

→ from continuity equation

$$A_1 v_1 = A_2 v_2$$

$$\boxed{v_1 = \frac{A_2}{A_1} v_2}$$

$$\therefore \cancel{\frac{v_2^2 - v_1^2}{2g}} =$$

$$\rightarrow \frac{v_2^2 - v_2^2\left(\frac{A_2}{A_1}\right)^2}{2g} = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\rightarrow \frac{v_2^2\left\{1 - \left(\frac{A_2}{A_1}\right)^2\right\}}{2g} = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\rightarrow v_2^2 = \frac{2g\Delta h\left(\frac{\rho_m}{\rho} - 1\right)}{\left\{1 - \left(\frac{A_2}{A_1}\right)^2\right\}}$$

velocity at throat
$$\boxed{V_2 = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \times \sqrt{2gh\left(\frac{\rho_m}{\rho}-1\right)}}$$

* For Discharge (Q) $m^3/sec$

$$Q = AV = A_2 V_2$$

$Q_T$ (Theoritical discharge)

$$= \frac{A_2}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \times \sqrt{2gh\left(\frac{\rho_m}{\rho}-1\right)}$$

$$\boxed{Q_T = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh\left(\frac{\rho_m}{\rho}-1\right)}}$$

⇒ $Q_{act} < Q_T$

→ Coefficient of discharge

$$C_d = \frac{Q_{act}}{Q_T}$$

∴ Actual discharge

$$\boxed{Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh\left(\frac{\rho_m}{\rho}-1\right)}}$$

$\rho_m > \rho$

→ If $\rho_m < \rho$

↳ $Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2g\Delta h\left(1 - \frac{\rho_m}{\rho}\right)}$

* $h_1^* - h_2^*$

or $= \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$ $\rho_m > \rho$

$h_1 - h_2$

~~If $h_1^*$~~

* In terms of specific gravity

$h_1 - h_2$

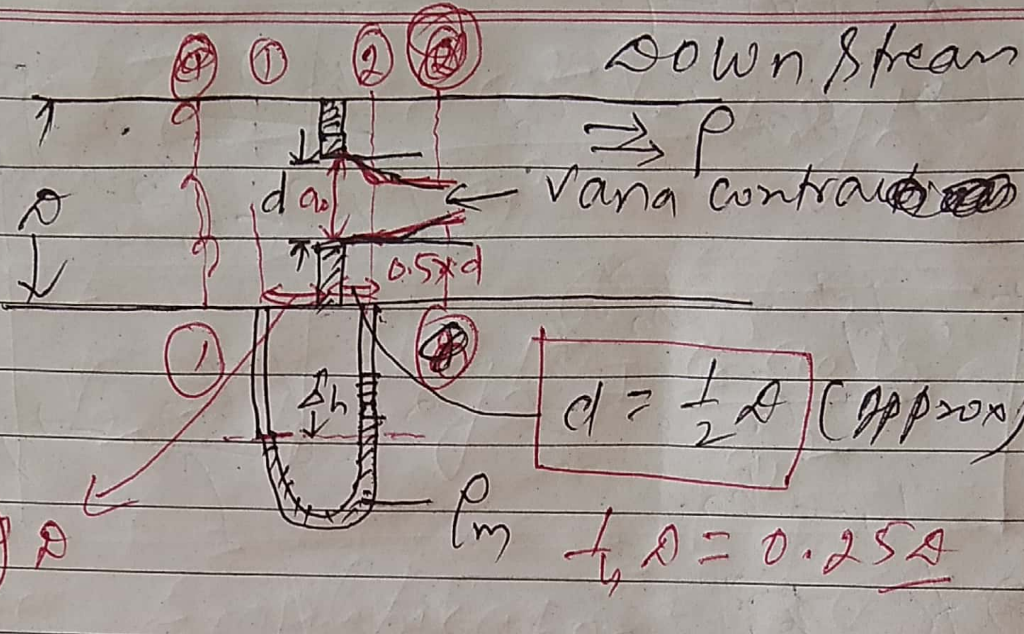
or $= \Delta h\left(\frac{(S.G)_m}{(S.G)} - 1\right)$

$h_1^* - h_2^*$

$(SG)_m > (S.G)$

→ If $(SG)_m < (S.G)$

# * Orifice meter




$d = \frac{1}{2}D$ (approx)

$\frac{1}{4}D = 0.25D$

## * Use of orifices

→ is to measure discharge, flow rate

## * Bernoullis equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\rightarrow \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

→ piezometric head

$$\rightarrow \boxed{h_1^* - h_2^* = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}}$$

$h^*$ = differential head

$$h^* = \frac{v_2^2 - v_1^2}{2g}$$

$$\boxed{v_2^2 = v_1^2 + 2gh^*} \quad \text{—— (I)}$$

* $$\boxed{h^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)}$$

* continuity of equation.

$$a_1 v_1 = a_2 v_2$$

$$\boxed{v_1 = \frac{a_2 v_2}{a_1}} \quad \text{—— (II)}$$

$$\boxed{v_2^2 = \frac{a_2^2 v_2^2}{a_1^2} + 2gh^*} \quad \text{—— (III)}$$

**NOTE** $a_0$ = Area of c/s of orifice

$C_c$ = coefficient of contraction

$$\boxed{C_c = \frac{a_2}{a_0}}$$

$$\boxed{a_2 = C_c \cdot a_0} \quad \text{—— (IV)}$$

from eqn (II) & (IV)

$$\boxed{v_1 = \frac{C_c a_0 V_2}{a_1}}$$

$$\boxed{V_2 = \sqrt{\frac{C_c^2 a_0^2 V_2^2}{a_1^2} + 2gh^*}}$$

Or

$$\boxed{V_2 = \sqrt{2gh^* + \frac{C_c^2 a_0^2 V_2^2}{a_1^2}}}$$

→ on simplification

$$\boxed{V_2 = \frac{\sqrt{2gh^*}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}}$$

$$\boxed{h^* = \Delta h \left(\frac{\rho_m}{\rho} - 1\right)}$$

* discharge

$$Q = a_2 V_2$$

$$\boxed{C_c = C_d \times \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}}$$

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## * Discharge Through Orifice

$$\boxed{Q = C_d \frac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh^*}}$$

* $h^* = \Delta h \left( \frac{(S.G)_m}{(S.G)} - 1 \right)$ Sp. gravity.

$\rho_m > \rho$

$(S.G)_m > (S.G)$

if $\rho_m < \rho$

$h^* = \Delta h \left( 1 - \frac{\rho_m}{\rho} \right)$

$h^* = \Delta h \left( 1 - \frac{(S.G)_m}{(S.G)} \right)$

Q/ An orifice meter with orifice diameter of 15 cm and pipe diameter 30 cm. the differential manometer gives a reading of 50 cm of mercury find the rate of flow of oil (sp. gravity = 0.9) if the coefficient of discharge is 0.64.

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Soln

$$Q = C_d \frac{a_1 a_0}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh^*} \quad \text{—(1)}$$

$C_d$ = 0.64

$a_1 = \frac{\pi}{4}(30)^2 = \frac{\pi}{4} \times 30^2 = 706.5\ cm^2$

$a_0 = \frac{\pi}{4}(15)^2 = \frac{\pi}{4} \times (15)^2 = 176.62\ cm^2$

(sp. gravity) = 13.6.
Manometer fluid.

$h^* = \Delta h\left(\frac{\rho_m}{\rho} - 1\right)$

put value in eqn (1)

$h^* = \Delta h\left(\frac{(S.G)_m}{(S.G)} - 1\right)$

$h^* = 50\left(\frac{13.6}{0.9} - 1\right) = 705.56\ cm.$

$$\boxed{Q = 137414.25\ \frac{cm^3}{sec.}}$$

$1\,\frac{m^3}{s} \longrightarrow 1\,m^3 = 1000\,m$

$10^6\,cm^3 = 1000$ litre $= 10^3$ litre.

$1000\,cm^3 = 1$ litres.

$1$ litre $= \frac{1}{1000}\,m^3$.

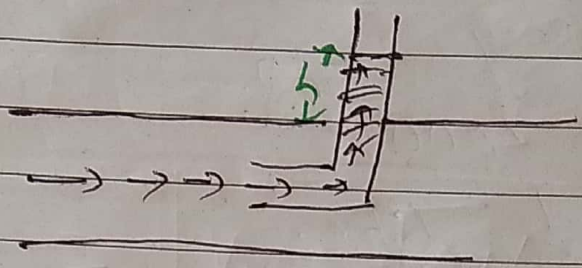
$1\,cm^3 = \frac{1}{1000}$ litre.

$$\theta = 0.64 \times \frac{706.5 \times 176.625}{\sqrt{(706.5)^2 - (176.625)^2}} \times \sqrt{2 \times 9.81 \times 205.56}$$

$$= \frac{117.656}{684.06} \times$$

# (3) Pitot tube

→ velocity measurement



Three type of pressure

(i) static pressure (Thermodynamic pressure)
(ii) Dynamic pressure → Kinetic Energy per unit volume
(iii) stagnation pressure.

$= \frac{1}{2} m v^2$

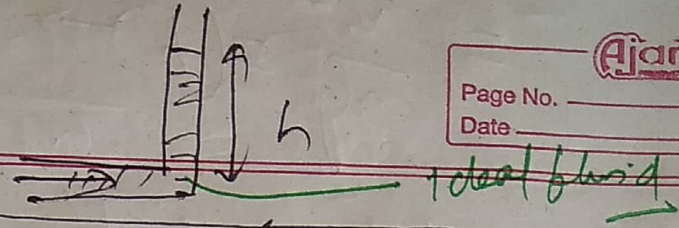
$\frac{K.E}{V} = \frac{\frac{1}{2} m v^2}{\frac{m}{\rho}}$

$\rho, V$
$V = \frac{m}{\rho}$

→ $\boxed{\text{Dynamic pressure} = \frac{1}{2} \rho v^2}$

(iii) Stagnation pressure →

→ When a fluid is moving K.E
K.E → pressure head.




$\boxed{K.E \rightarrow \text{pressure head}}$

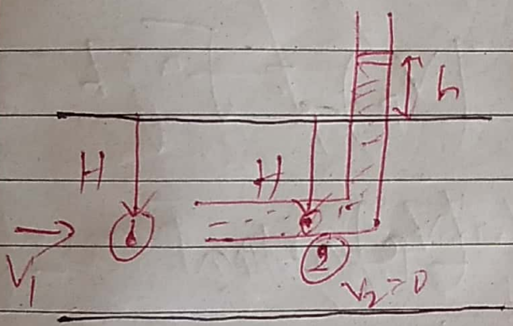
$\downarrow$

in an isentropic process.

→ Reversible adiabatic process

$\downarrow$

Isentropic.




* Applying Bernoulli's equation

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

→ $\frac{P_1}{\rho g}$ = pressure head above 1 = $H$

→ $\frac{P_2}{\rho g}$ = pressure head above 2 = $H+h$

→ $\cancel{H} + \frac{V_1^2}{2g} = (\cancel{H}+h) + 0 \quad \Big| \because V_2 = 0$

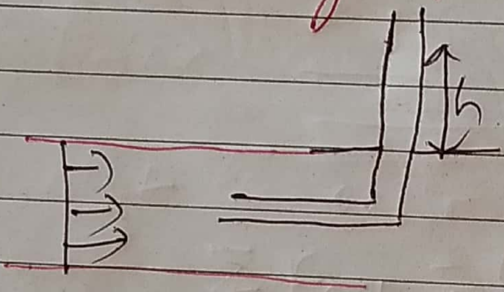
the bend tube.

↓

pitot tube

$$\Rightarrow \boxed{V_1 = \sqrt{2gh}}$$

+ Velocity of Real fluids




$(V_P)_T = \sqrt{2gh}$

$(V_P)_{actual}$

$$\therefore \boxed{\frac{(V_P)_{actual}}{(V_P)_T} = C_V}$$

$$\boxed{V_{act} = C_V \sqrt{2gh}}$$

## * Momentum equation



→ **Newton's second law**

$$f = ma \quad (\text{Law of Net force})$$

$$a = \frac{dv}{dt}$$

$$\therefore \boxed{F = m\frac{dv}{dt}}$$

$$\cancel{F = \frac{mdv}{dt}}$$

$$f = \frac{d\,mv}{dt}$$

$mv$ = Momentum.

$\frac{d(mv)}{dt}$ = Rate of change of momentum.

$$\therefore \boxed{F\,dt = d(mv)}$$

Impulse = change in Momentum

→ Impulse Momentum Theorem

$$F \cdot dt = d(mv)$$

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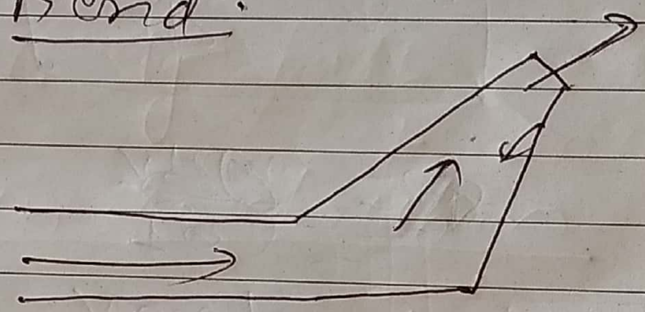
$$F = \frac{d(mv)}{dt}$$

Net force = Rate of change of momentum

## * Application of Momentum Equation →

→ pipe → Bend.



→ $\Sigma F_{ext}$ = Change in momentum
or
Rate of change in momentum.

→ Momentum → m v → m/sec.
↓
kg

⇒ Rate of change of momentum.

$$= \frac{kg\ m/sec}{sec} = \frac{kg\ m}{sec^2}$$

$$= \frac{kg\ (m)}{s} \times \frac{m\ (V)}{s}$$

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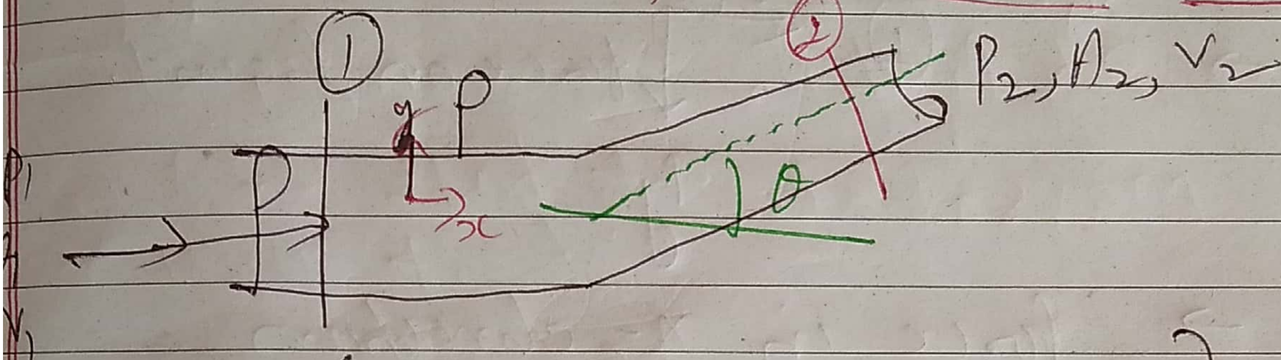
= Mass flow rate × Velocity

= Rate of change of Momentum

## * Application of momentum Equation




→ liquid is incompressible
→ inviscide

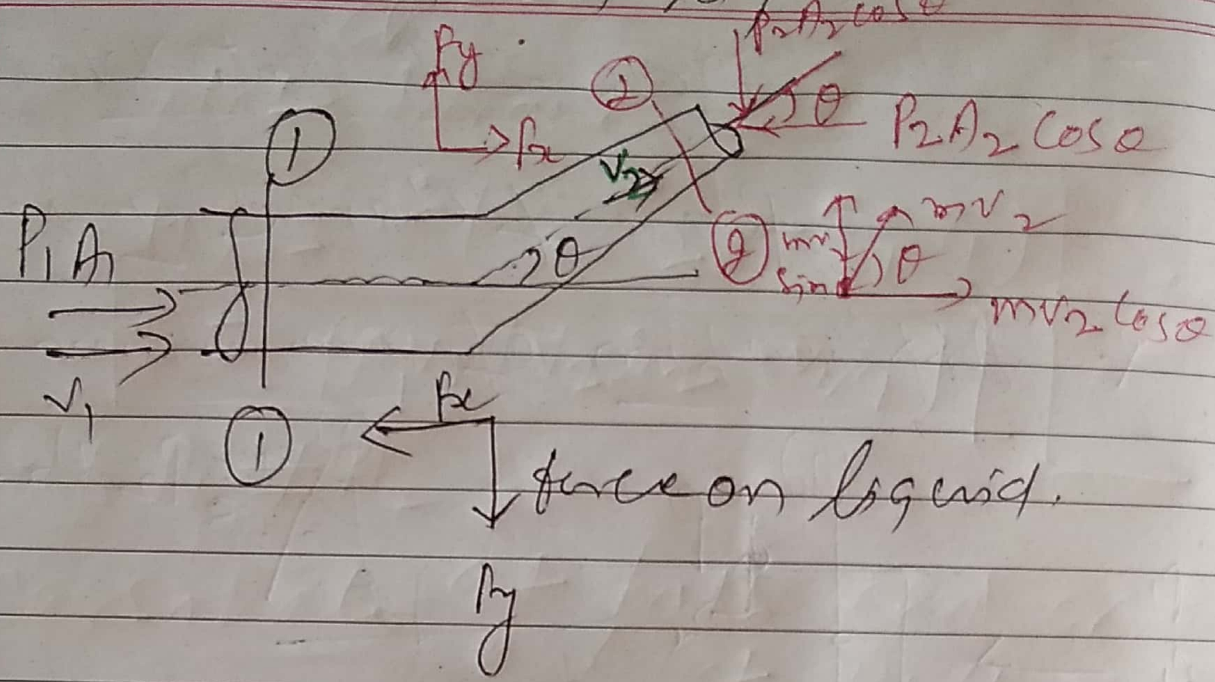
Ideal fluid.

* $F_x$ = force exerted by liquid on nozzel or pipe bend in x-direction.

→ force exerted by Nozzle or pipe bend on liquid = $-F_x$

→ $F_y$ = force exerted by liquid on pipe bend

→ $\cdot - F_y$ = Force exerted by pipe bend on liquid.




→ Force in x-direction

→ Net force in x-direction

$$= P_1A_1 - P_2A_2\cos\theta - F_x$$

→ Rate of change of Momentum

$$= \dot{m}(V_2\cos\theta - v_1)$$

$$= \rho A_1 \times v_1$$

$$= \rho A_1 v_1 (v_2\cos\theta - v_1)$$

$A v_1 = \frac{m^3}{s}$

$\rho A v_1 = \frac{kg}{m^3} \times \frac{m^3}{sec}$

$\rho A_2 v_2 = \frac{kg}{s}$

* Force in y-direction

→ Net force in y-direction,

$$0 - P_2A_2\sin\theta - F_y$$

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→ Rate of change of Momentum

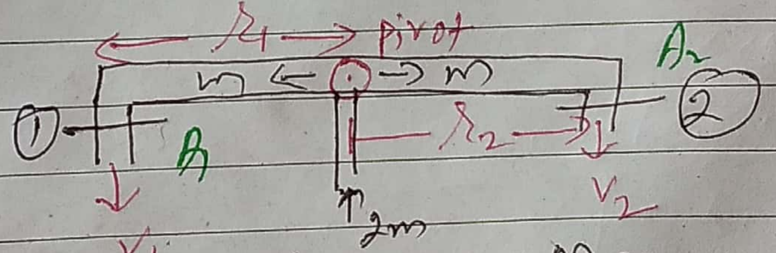
$$= \dot{m} v_2 \sin\theta - 0$$

* Resultant force on pipe bend.

$$= \sqrt{F_x^2 + F_y^2}$$

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

* **Moment of Momentum**




$A_1 = A_2$ (let)

Moment of momentum at ① $= m v_1$

$= m v_1 \cdot r_1$

Moment of Momentum at section ② $= m v_2$

$= m v_2 r_2$

→ Net Torque = Rate of change of Moment of Momentum

→ Resulting Torque
or
Net torque

$(T) = \dot{m}(V_2 r_2 - V_1 r_1)$

$$\boxed{T = \rho A_1 V_1 (V_2 r_2 - V_1 r_1)}$$

Clockwise moments +ve
Anticlockwise -ve

→ If $A_1 \neq A_2$

Mass flow Rate at both end or outlet will not be same.

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