We studied, Under biaxial state of stress, there are two principal stress one at θ_1

and at $\theta_2 = \theta_1 \pm 90^\circ$

Now, magnitude of principal stresses

$$(\sigma_n)_{\theta=\theta_1} = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}\frac{(\sigma_x + \sigma_y)}{2} \cos 2\theta_1 + \tau_{xy} \sin 2\theta_1$$

$$= \_\_\_\_ \text{ MPa}$$

$$(\sigma_n)_{\theta=\theta_1} + (\sigma_n')_{\theta=\theta_2} = \sigma_x + \sigma_y$$

$$\therefore (\sigma_n')_{\theta=\theta_2} = \sigma_x + \sigma_y - (\sigma_n)_{\theta=\theta_1}$$

Out of these two values, major principal stress is that in which magnitude is larger.

σ_1 = major principal stress

= Max normal stress

= larger of $[(\sigma_n)_{\theta=\theta_1}$ & $(\sigma_n')_{\theta=\theta_2})$

(only magnitude is considered)

σ_2 = Minor principal stress

= Min normal stress

= smaller of $\left[(\sigma_n)_{\theta=\theta_1}\right.$ and

$\left.(\sigma_n')_{\theta=\theta_2}\right]$

$$\boxed{\sigma_1 + \sigma_2 = \sigma_x + \sigma_y}$$

__NOTE__ → while deciding principal stresses only magnitude of principal stress should be considered.

Maximum shear stress plane and Max shear stress

On max^m shear stress plane, shear stress is maximum, but normal stress is ~~none~~ non-zero (ie some amount of σ_n)

Location

$$\frac{d}{d\theta}(\tau_s) = 0$$

$$\frac{d}{d\theta}\left[-\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta\right] = 0$$

~~tan 2θ~~

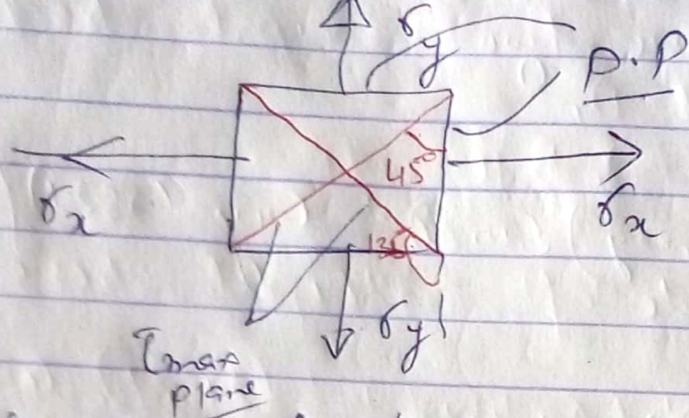
$$\Rightarrow \left[-\frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta\right]2 - (\tau_{xy}\sin 2\theta)\cdot 2 = 0$$

$$\Rightarrow \tan 2\theta = -\frac{(\sigma_x-\sigma_y)}{2\tau_{xy}}$$

$$\Rightarrow \boxed{\tan 2\theta = -\frac{1}{\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)}}$$

Scanned with CamScanner

Now - Consider the following case



$\Rightarrow$ only normal stresses are acting, ie σ_x & σ_y

$\tau_{xy} = 0$

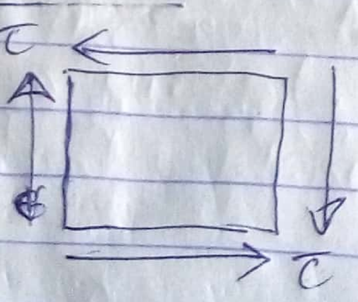
$$\tan 2\theta = \frac{-\sigma_x - \sigma_y}{2\tau_{xy}}$$

$\therefore \tan 2\theta = \infty$

$2\theta = 90°$ or $270°$

$\therefore \theta = 45°$ or $135°$

(II) Pure Shear



Here $\tan 2\theta = 0$

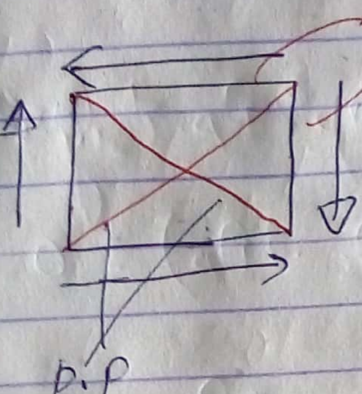
$2\theta = 0$ or $180°$

$\theta = 0$ or $90°$

τ_{max} plane or plane of pure shear

or plane of zero normal stress

~~valid on~~ valid only in this case



Again τ_{max} plane are mutually $\perp r$ to each other

In plane $\tau_{max} = (\tau_s)_{\theta_3 \text{ or } \theta_4}$

Suppose θ_s = orientation of the plane of maximum positive or negative shear stresses & θ_p for maximum normal stress

then $\tan 2\theta_s = -\dfrac{(\sigma_x - \sigma_y)}{2\tau_{xy}} = -\dfrac{1}{\left(\dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$

$= -\dfrac{1}{\tan 2\theta_p} = -\cot 2\theta_p$

$$\Rightarrow \frac{\sin 2\theta_s}{\cos 2\theta_s} + \frac{\cos 2\theta_p}{\sin 2\theta_p} = 0$$

$$\Rightarrow \sin 2\theta_s \cdot \sin 2\theta_p + \cos 2\theta_s \cdot \cos 2\theta_p = 0$$

$$\Rightarrow \cos(2\theta_s - 2\theta_p) = 0$$

There fore $2\theta_s - 2\theta_p = \pm 90^\circ$

$$\boxed{\theta_s = \theta_p \pm 45^\circ}$$

~~This equation shows that the planes of maximum shear stress occur at the principal planes.~~

This equation shows that the planes of maximum shear stress occur at 45° to the principal planes.

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—(A)}$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{—(B)}$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{—(C)}$$

From (A) & (C)

$$\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2} = \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—(1)}$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{—(2)}$$

squaring both side of eq^n (1) & (2) then adding (1) & (2) we get

$$\left\{\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right\}^2 + \tau_{x'y'}^2$$

$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)^2\left\{\cos^2 2\theta + \sin^2 2\theta\right\} + (\tau_{xy})^2\left\{\sin^2 2\theta + \cos^2 2\theta\right\}$$

$$\Rightarrow \boxed{\left\{\sigma_{x'} - \left(\frac{\sigma_x+\sigma_y}{2}\right)\right\}^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Setting

$$\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

we get

$$\boxed{\left(\sigma_{x'} - \sigma_{avg}\right)^2 + \tau_{x'y'}^2 = R^2}$$

which is the equation of a circle of radius R centred at the point C of abscissa σ_{avg} and ordinate 0