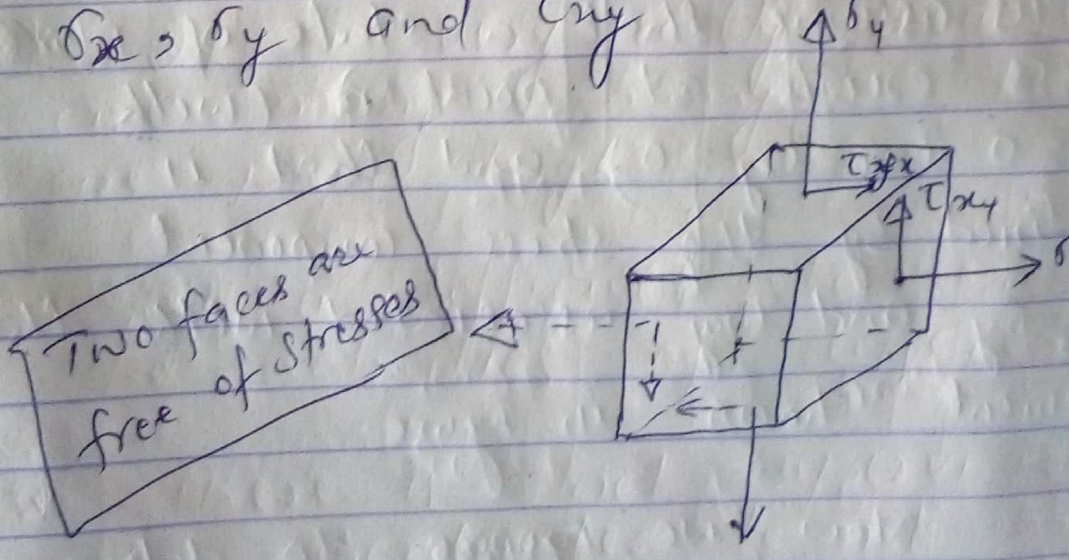
PLANE STRESS

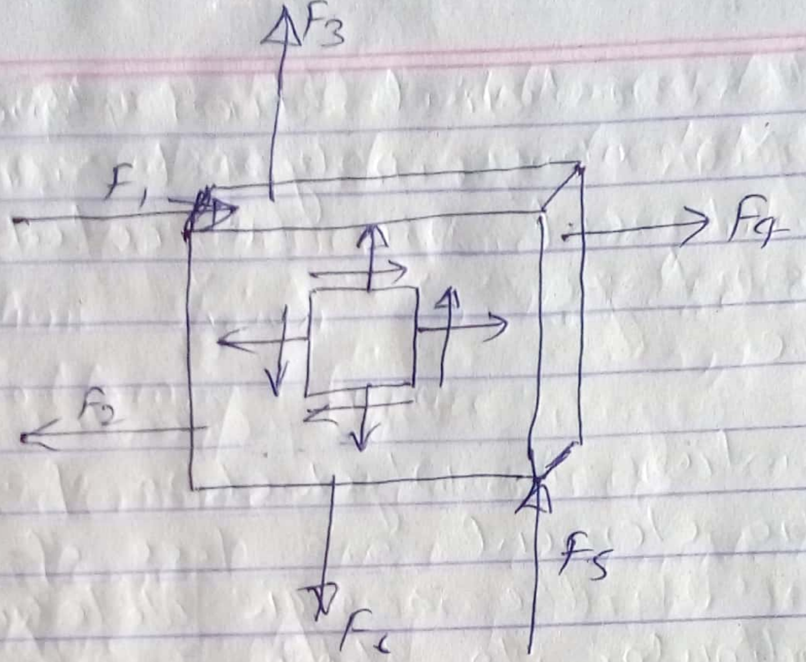
When the material is in plane stress in the XY plane, only the x, and y faces of the element are subjected to stresses

i.e

A situation in which two of faces of the cubic element are free of any stress. If the z-axis is chosen perpendicular to these faces, we have $\sigma_z = \tau_{zx} = \tau_{zy} = 0$ and the only remaining stress components are σ_x, σ_y and τ_{xy}



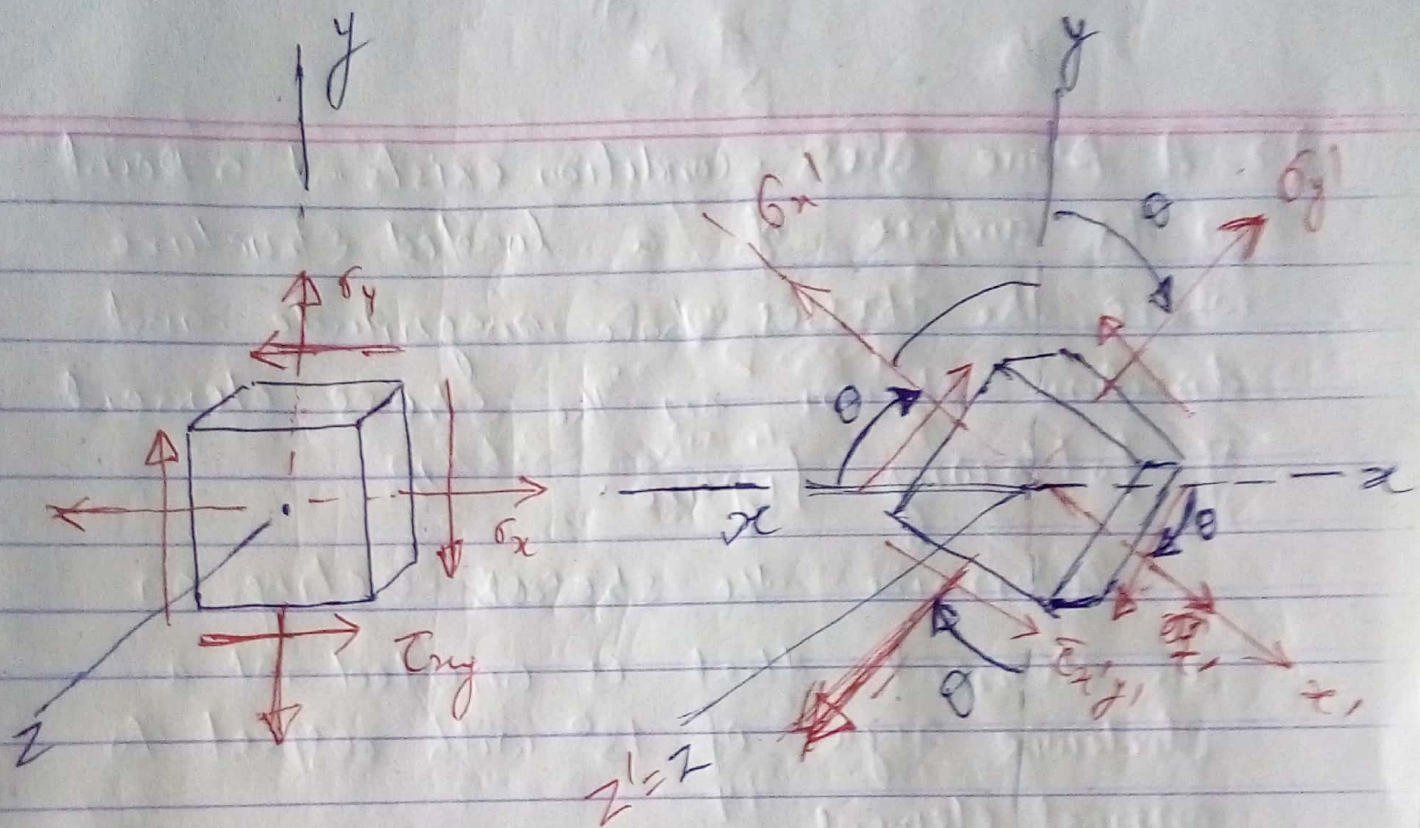
Such a situation occurs in a thin plate subjected to forces acting in mid plane of the plate.



TRANSFORMATION OF PLANE STRESS

Let us assume that a state of plane stress exists at point Q (with $\sigma_z = 0$ $\tau_{zx} = \tau_{zy} = 0$) and that is defined by the stress component σ_x, σ_y and τ_{xy} associated with the element shown in figure.

Now, we propose to determine the stress component σ_x', σ_y' and $\tau_{x'y'}$ associated with the element after it has been rotated through an angle θ about the z-axis



We earlier derive expression for

$$\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{— (A)}$$

$$\sigma_y' = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{— (B)}$$

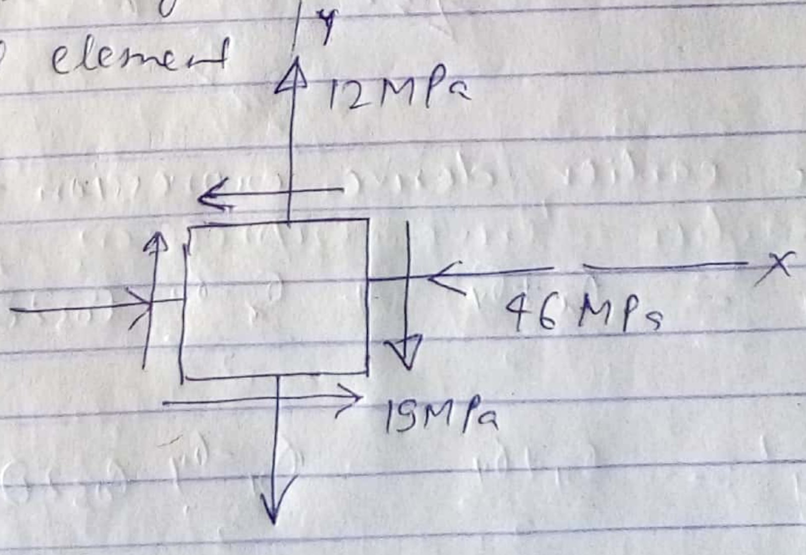
$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{— (C)}$$

From eqn (A) & (B)

we have $\boxed{\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y}$

Q→ A plane stress condition exists at a point on the surface of a loaded structure where the stresses have magnitude and directions shown on the stress element of following figure.

Determine the stresses acting on an element that is oriented at a clockwise angle of 15° with respect to original element



Soln→ Here $\sigma_x = -46\,MPa$

$\sigma_y = 12\,MPa$

$\tau_{xy} = 19\,MPa$

$\theta = 15^\circ$

$$\therefore \sigma_{x}' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-46+12}{2} + \left(\frac{-46-12}{2}\right)\cos 30^\circ + 19\sin 30^\circ$$

$$= \cancel{-46}\ \frac{-34}{2} + \frac{-58}{2}\cdot\frac{\sqrt{3}}{2} + \frac{19}{2}$$

$$= -17 - (14.5)\sqrt{3} + 9.5$$

$$= -32.6\ MPa$$

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-46+12}{2} - \left(\frac{-46-12}{2}\right)\cos 30^\circ - 19\cdot\sin 30^\circ$$

$$= -17 + (19.5)\sqrt{3} - 9.5$$

$$= -26.5 + (14.5)\sqrt{3}$$

$$= -1.38\ MPa$$

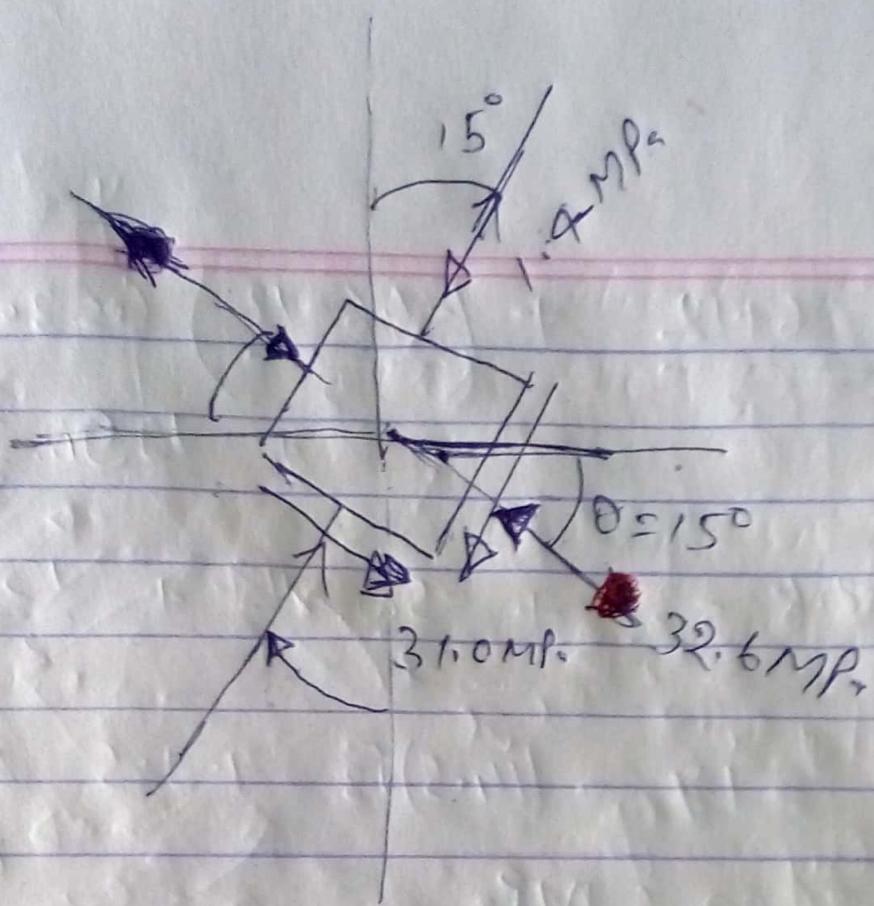
$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-46-12}{2}\right)\sin 30^\circ + 19\cos 30^\circ$$

$$= +\frac{29}{\cancel{4}2} + \frac{19\sqrt{3}}{2} = 14.5 + 9.5\sqrt{3}$$

$$= \cancel{0.25+0.95\sqrt{3}} = 30.95\,MPa$$

Scanned with CamScanner



15°
1.4 MPa
θ=15°
31.0 MPa
32.6 MPa