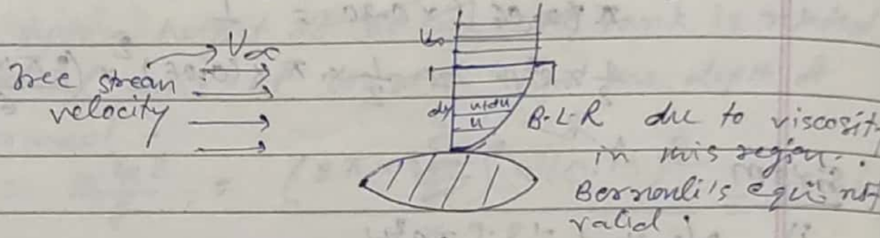
$\therefore V(AED) - V(BCE) = 8.09\times10^{-3} - 1.41\times10^{-3} = 2.68\times10^{-3}\,m^3$

$\text{Vol. left} = 3.39\times10^{-3} - 2.68\times10^{-3}$
$= 0.71\times10^{-3}$ Ans

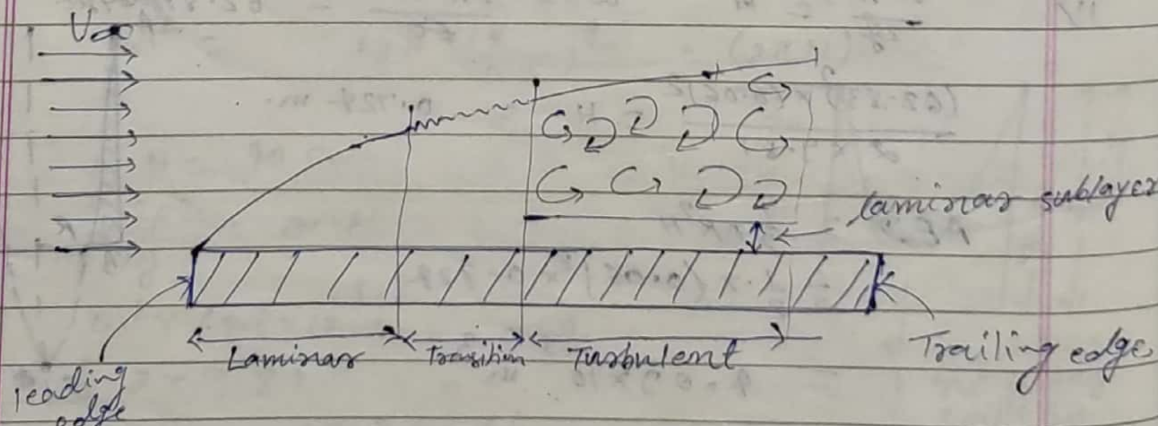
BOUNDRY LAYER THEORY

BPSC

This theory was proposed by Prandtl in 1904



→ When a real fluid flows passed a solid object a fluid particle on the surface will have same velocity as that of the surface because of no slip condition. If the boundary is static or the object is static the fluid particles will also have zero velocity. Away from the boundary the vel. gradually increases and vel. gradient du/dy exist in a region close to the boundry this region is known as boundry layer region. These vel. gradient will give rise to shear stress and hence the flow is viscous in boundry layer region.



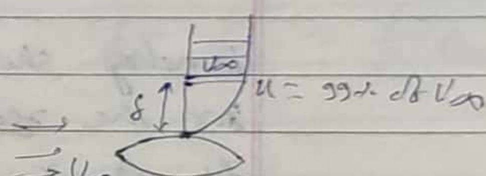
When a real fluid flows passed a flat plate the velocity at the leading edge is zero. And the retardation of fluid particles increases as more area of the plate is exposed to the flow.

and hence the boundary layer thickness increases as the distance from the leading edge increases. upto certain distance from leading edge the flow in the boundary layer is found to be laminar and as the laminar boundary layer grows instability occurs and the flow changes from laminar to turbulent through transition. It is found that even in turbulent boundary layer region close to the plate the flow is laminar this region is known as laminar sublayer region. and this exists in turbulent boundary layer region.

Boundary layer thickness (δ) :→

→ or Nominal boundary layer thickness :→

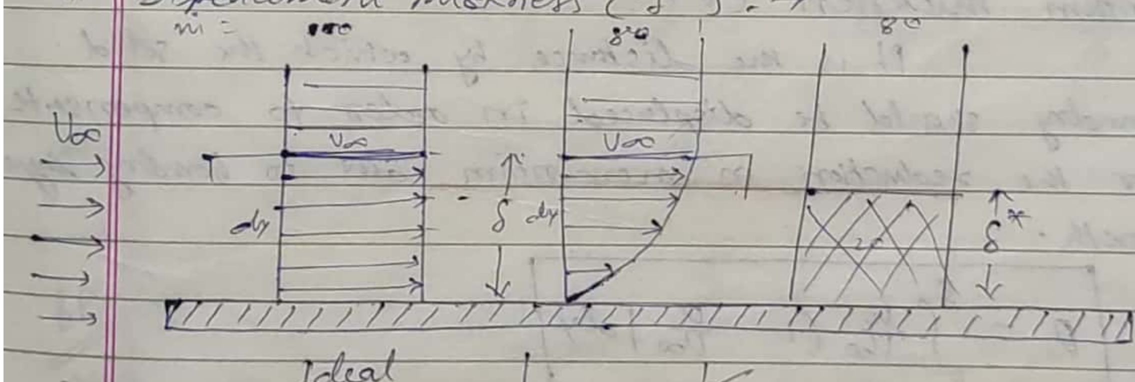
It is the distance from the boundary to the point in y-direction where the velocity is 99% of free stream vel (V_∞)

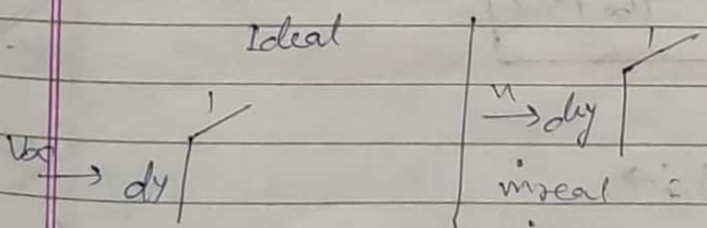


at $y = \delta$, $U = 0.99 V_\infty$
$\approx V_\infty$

Note :- for all calculation purposes $u = V_\infty$ at $y = \delta$

Displacement thickness (δ^*) :→





$\dot{m}_{real} = \rho \times dy \times 1 \times u$

$\dot{m}_{real} = \rho u dy$

$\dot{m}_{ideal} = \rho\, dy \times 1 \times V_\infty$

$\dot{m}_{ideal} = \rho V_\infty dy$

reduction in mass flow rate $= \dot{m}_{ideal} - \dot{m}_{real}$

$$= \rho U_\infty dy - \rho u dy$$

$$= \rho (U_\infty - u) dy$$

Total reduction in mass flow rate $= \int_0^\delta \rho (U_\infty - u) dy$

in ideal case:

$U_\infty \rightarrow$ δ^*

$$\dot{m} = \rho \delta^* \times 1 \times U_\infty$$

$$= \rho \delta^* U_\infty$$

$$\therefore \rho \delta^* U_\infty = \int_0^\delta \rho (U_\infty - u) dy$$

ES' derivation

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$

It is the distance by which the solid boundry should be displaced in order to compensate for the reduction in mass flow rate due to boundry layer growth.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

Momentum thickness (θ) :→

It is the distance by which the solid boundry should be displaced in order to compensate for the reduction in momentum due to boundry layer growth.

$$\boxed{\theta = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u}{U_\infty}\right) dy}$$

Energy thickness (δ_E) :→

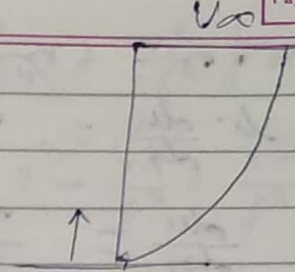
It is the distance by which the boundry should be displaced in order to compensate for the reduction in kinetic energy due to boundry layer growth

$$\delta_E = \int_0^\delta \frac{u}{U_\infty} \left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

$$\delta_E = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

V.V.I

Bondry conditions :→



i) $y = 0, \quad u = 0$

2) $y = \delta, \quad u = U_\infty$

3) $y = \delta, \quad \frac{du}{dy} = 0$ $\qquad \tau_0 = \mu \frac{du}{dy}\Big|_{y=0}$ at the surface

4) At $x = 0, \quad \delta = 0$

ES 3 times

Q:→ The velocity distribution in a bondry layer is given by $\frac{u}{U_\infty} = \frac{y}{\delta}$ find displacement thickness δ^* and momentum thickness θ.

Sol:→
$$\delta^* = \int_0^{\delta}\left(1 - \frac{y}{\delta}\right) dy$$
$$= \left[y - \frac{y^2}{2\delta}\right]_0^{\delta} = \delta - \frac{\delta^2}{2\delta} = \frac{\delta}{2} \text{ Ans}$$

$$\theta = \int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$
$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^{\delta} = \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2}$$
$$= \frac{\delta}{2} - \frac{\delta}{3} = \frac{3\delta - 2\delta}{6} = \frac{\delta}{6} \text{ Ans}$$

Note:→ shape factor of BL (H) $= \frac{\delta^*}{\theta}$
$$= \frac{\delta}{2} \times \frac{6}{\delta} = 3$$

BPS

Note:- The shape factor of a BL (H) $= \frac{\delta^*}{\theta} = \frac{\text{disp. thickness}}{\text{Momentum thickness}}$

Larger the value of shape factor bondry layer seperation is easy.

Q:→ The shear stress distribution in a laminar bondry layer is given by $\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$. find displacement thickness and momentum thickness in terms of δ.

Date / /
Page

$$\tau = \tau_0\left(1-\frac{y}{\delta}\right)$$

$$\mu\cdot\frac{du}{dy} = \tau_0\left(1-\frac{y}{\delta}\right)$$

$$\frac{du}{dy} = \frac{\tau_0}{\mu}\left(1-\frac{y}{\delta}\right)$$

$$\int_0^{u_\infty} du = \int_0^{\delta}\frac{\tau_0}{\mu}\left(1-\frac{y}{\delta}\right)dy$$

$$u_\infty = \frac{\tau_0}{\mu}\left[y-\frac{y^2}{2\delta}\right]_0^{\delta}$$

$$= \frac{\tau_0}{\mu}\left[\delta-\frac{\delta^2}{2\delta}\right)$$

$$= \frac{\tau_0}{\mu}\left(\delta-\frac{\delta}{2}\right) = \frac{\tau_0}{\mu}\cdot\frac{\delta}{2}.$$

$$\int_0^{u} du = \int_0^{y}\frac{\tau_0}{\mu}\left(1-\frac{y}{\delta}\right)dy$$

$$u = \frac{\tau_0}{\mu}\left[y-\frac{y^2}{2\delta}\right]_0^{y}$$

$$u = \frac{\tau_0}{\mu}\left[y-\frac{y^2}{2\delta}\right]$$

$$\frac{u}{u_\infty} = \frac{\cancel{\frac{\tau_0}{\mu}}\left(y-\frac{y^2}{2\delta}\right)}{\cancel{\frac{\tau_0}{\mu}}\cdot\frac{\delta}{2}} = \frac{2}{\delta}\left(y-\frac{y^2}{2\delta}\right)$$

$$= \frac{2}{\delta}y-\frac{y^2}{\delta^2}$$

$$\delta^* = \int_0^{\delta}\left(1-\frac{2}{\delta}y+\frac{y^2}{\delta^2}\right)dy$$

$$= \left[y-\frac{\cancel{2}}{\delta}\cdot\frac{y^2}{\cancel{2}}+\frac{y^3}{3\delta^2}\right]_0^{\delta}$$

$$= \delta-\frac{1}{\delta}\cdot\delta^2+\frac{\delta^3}{3\delta^2}$$

$$\cancel{\delta}-\cancel{\delta}+\frac{\delta}{3} = \frac{\delta}{3}\ \text{Ans}$$

Scanned by CamScanner

Date / /
Page

$$\theta = \int \left(\frac{2}{\delta} y - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2}{\delta} y + \frac{y^2}{\delta^2}\right) dy$$

$$= \int \left(\frac{2}{\delta} y - \frac{4}{\delta^2} y^2 + \frac{2}{\delta^3} y^3 - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$= \int_0^{\delta} \left(\frac{2}{\delta} y - \frac{5}{\delta^2} y^2 + \frac{4}{\delta^3} y^3 - \frac{y^4}{\delta^4}\right) dy$$

$$= \left[\frac{2}{\delta}\cdot\frac{y^2}{2} - \frac{5}{\delta^2}\cdot\frac{y^3}{3} + \frac{4}{\delta^3}\cdot\frac{y^4}{4} - \frac{y^5}{5\delta^4}\right]_0^{\delta}$$

$$= \frac{\delta^2}{\delta} - \frac{5}{3}\frac{\delta^3}{\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}$$

$$= 2\delta - \frac{5}{3}\delta - \frac{\delta}{5} = \frac{30\delta - 25\delta - 3\delta}{15} = \frac{2\delta}{15} \text{ Ans}$$

Si/s M

$$u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

$y = \delta, \quad u = u_\infty$

$$u_\infty = \frac{\tau_0}{\mu}\cdot\frac{\delta}{2}$$

$\frac{u}{u_\infty} = ?$ $\quad \delta^* = ?$ $\quad \theta = ?$

Von karman momentum integral Equation :→

Assumptions:-

i) steady flow ii) Two dimensional flow iii) incompressible flow

iv) Pressure gradient in the direcn of flow $\left(\frac{dp}{dx}\right) = 0$

$$\boxed{\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}}$$

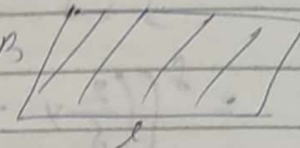
where τ_0 = Shear stress on the surface of the plate

θ = momentum thickness.

→ **Drag force (F_d)** :- The force exerted in a direction relative to parallel motion is known as drag force at zero angle of incidence drag force is due to shear force

AVERAGE DRAG COEFFICIENT (C_D) :→

$$\boxed{C_D = \frac{F_D}{\frac{1}{2} \rho A V_\infty^2}}$$



$A = Bl$

$A = B.l$

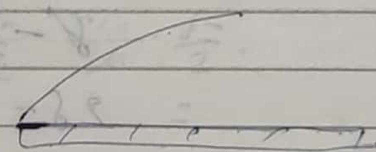
Local skin friction coefficient (C_{fx}) :-
(or local drag coefficient) :

$$\boxed{C_{fx} = \frac{\tau_0}{\frac{1}{2} \rho V_\infty^2}}$$

C_{fx} gives shear stress at local distance

Reynolds no. (Re) :→

$$Re = \frac{\rho V L}{\mu}$$



$Re < 5\times10^5$ (laminar)

$$\boxed{Re_x = \frac{\rho V_\infty x}{\mu}}$$

$$\boxed{Re_L = \frac{\rho V_\infty L}{\mu}}$$

If $Re_L < 5\times10^5$ (laminar)

where x = distance from the leading edge
for a laminar flow in the boundary layer
Re must be less than 5×10^5.

$Re = \frac{\rho V L}{\mu}$ for circular L = Dia.
in general $L = \frac{4\,\text{Area}}{\text{perimeter}}$

Note :→ for noncircular ducts the characteristic length
$L = D_{eq} = \frac{4A}{P}$ ~~ie~~

for rectangular $\frac{4A}{P} = \frac{4 \times ab}{2(a+b)} = \frac{2ab}{a+b}$

Date / /
Page

BPS (U)

Q.→ for a velocity profile in laminar boundary layer $\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$. find

i) Boundary layer thickness

ii) Shear stress on the surface of the plate (τ_0)

iii) Drag force

iv) Average drag coefficient C_D in terms of Re

soln→

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}$$

$$\tau_0 = \rho U_\infty^2 \frac{d\theta}{dx}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$= \int_0^\delta \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \frac{3y}{2y} + \frac{y^3}{2\delta^3}\right) dy = \frac{39\delta}{280}$$

$$= \int_0^\delta$$

$$\tau_0 = \rho U_\infty^2 \cdot \frac{d}{dx}\left(\frac{39\delta}{280}\right) = \frac{39}{280} \rho U_\infty^2 \cdot \frac{d\delta}{dx} \quad \text{—(i)}$$

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \cdot U_\infty \left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right]$$

$$\tau_0 = \mu U_\infty \left(\frac{3}{2\delta} - \frac{3(0)^2}{2\delta^3}\right) = \frac{3\mu U_\infty}{2\delta} \quad \text{at } y=0 \quad \text{—(ii)}$$

from (i) & (ii)

$$\frac{39}{280} \rho U_\infty^2 \cdot \frac{d\delta}{dx} = \frac{3\mu U_\infty}{2\delta}$$

$$\frac{d\delta}{dx} = \frac{3\mu}{2\delta} \times \frac{280\ 140}{39\ \rho U_\infty\ 13}$$

$$\delta\, d\delta = \frac{140}{13} \frac{\mu}{\rho U_\infty} dx$$

$$\int_0^\delta \delta\, d\delta = \frac{140}{13} \frac{\mu}{\rho U_\infty} \int_0^x dx$$

$$\frac{\delta^2}{2} = \frac{140}{13} \frac{\mu}{\rho U_\infty} x$$

$Re = \frac{\rho V_\infty x}{\mu}$

Date / /
Page

$$\delta^2 = \frac{280}{13}\frac{\mu}{\rho V_\infty}\cdot x$$

$$\delta^2 = \frac{280}{13}\cdot\frac{\mu x}{\rho V_\infty}\cdot\frac{x}{x}$$

$$\delta^2 = \frac{280}{13}\cdot\frac{x^2}{\left(\frac{\rho V_\infty x}{\mu}\right)} = \frac{280}{13}\cdot\frac{x^2}{Re_x}$$

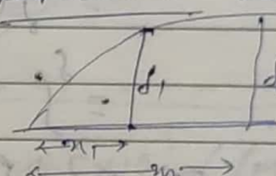
$$\boxed{\delta = \frac{4.64\,x}{\sqrt{Re_x}}}$$

$\delta^2 = \frac{280}{13}$

$\delta \propto \frac{x}{\sqrt{x}}$

$\delta \propto \frac{x}{\sqrt{x}}\sqrt{x}$

$\cancel{\delta\sqrt{x} \propto 1}$ $\quad \delta \propto \sqrt{x}$



$\boxed{\delta \propto \sqrt{x}}$ Ist objective question

ii) from equ (2)

$$\tau_0 = \frac{3\,\mu V_\infty}{2\,\delta}$$

$$= \frac{3\,\mu V_\infty}{2\times\frac{4.64\,x}{\sqrt{Re_x}}}$$

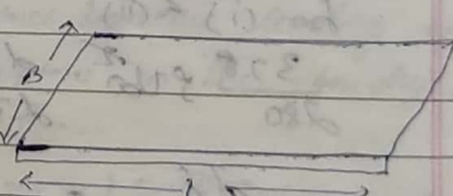
$\frac{3}{2\times 4.64} = 0.323$

$$= \frac{3}{2\times 4.64}\,\frac{\mu V_\infty}{x}\cdot\sqrt{Re_x}$$

$$\boxed{\tau_0 \propto \frac{1}{\sqrt{x}}}$$ → 2nd objective question

iii) $dF_D = \tau_0\cdot B\,dx$

$$= \frac{3}{2\times 4.64}\,\frac{\mu V_\infty}{x}\,\sqrt{Re_x}\cdot B\,dx$$



$$= \frac{3}{2\times 4.64}\cdot\frac{\mu V_\infty}{x}\cdot\frac{\sqrt{\rho V_\infty x}}{\sqrt{\mu}}\cdot B\,dx$$

$$\int dF_D = 0.323\,\frac{\mu V_\infty^{3/2}\sqrt{\rho}}{\sqrt{\mu}}\cdot B\int_0^L\frac{1}{\sqrt{x}}\cdot dx$$

$$= 0.323\cdot\mu V_\infty\sqrt{\frac{\rho V_\infty}{\mu}}\cdot B\cdot\left[2\sqrt{x}\right]_0^L$$

$$F_D = 0.646\,\mu V_\infty\sqrt{\frac{\rho V_\infty}{\mu}}\,B\sqrt{L}$$

$$\boxed{F_D = 0.646 \cdot \mu U_\infty B \sqrt{Re_L}}$$

iv) $$C_D = \frac{F_D}{\frac{1}{2}\rho A U_\infty^2} = \frac{0.646\,\mu U_\infty B\sqrt{Re_L}}{\frac{1}{2}\rho A U_\infty^2}$$

$A = BL$

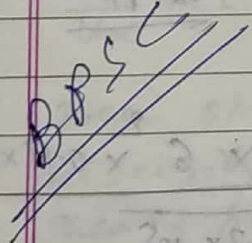
$$= 0.646\,\mu B \cdot \sqrt{\frac{\rho U_\infty L}{\mu}} \cdot \frac{1}{\frac{1}{2}\rho B L U_\infty^2}$$

$$= 1.292\,\frac{\sqrt{\mu}}{\sqrt{\rho L U_\infty}}$$

$$= \frac{1.292}{\sqrt{\frac{\rho U_\infty L}{\mu}}} = \frac{1.292}{\sqrt{Re_L}}$$

→ when the velocity profile is not given use the following equations.

Balacius



	Laminar	turbulent
	$\delta = \frac{5x}{\sqrt{Re_x}}$	$\delta = \frac{0.576x}{(Re_x)^{1/5}}$
	$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.059}{(Re_x)^{1/5}}$
	$C_D = \frac{1.328}{\sqrt{Re_L}}$	$C_D = \frac{0.074}{(Re_L)^{1/5}}$

Q.) Calculate the drag force on the plate 0.15m wide and 0.45m long placed in a stream of oil having a free stream velocity of 6 m/sec also. find the thickness of boundary layer and shear stress at the trailing edge. take density of oil as ~~920~~ 925 kg/m³. kinematic viscosity = 9×10^{-5} m²/sec.

Date / /
Page

Sol:- $Re_L = \frac{\rho V_\infty L}{\mu}$

$= \frac{V_\infty L}{(\mu/\rho)}$

$= \frac{6 \times 0.45}{9 \times 10^{-5}}$

$= \frac{\cancel{2.70}}{9} \times 10^{+5}$

$= 0.3 \times 10^{+5} < 5 \times 10^5$

So laminar

Now
$C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^2 A}$

$\frac{1.328}{\sqrt{Re_L}} = \frac{F_D}{\frac{1}{2}\frac{\rho V_\infty L}{\mu} \cdot \mu \cdot V_\infty}$

$F_D = \cancel{1.3 \cdot 2}\ \frac{1.328 \times \frac{1}{2}\rho V_\infty^2 A}{\sqrt{Re_L}}$

$= \frac{1.328}{2} \times \frac{925 \times 6^2 \times 0.15 \times 0.45}{\sqrt{0.3 \times 10^5}}$

$= 8.6\ N$

Again from laminar,
$\cancel{\delta = \frac{5x}{\sqrt{Re_x}}} = \frac{5x}{\ }$

This is the drag force on one side of the plate.

Date / /
Page

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{\tau_0}{\frac{1}{2}\rho V_\infty^2}$$

$$\frac{0.664}{\sqrt{Re_L}} = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2}$$

$$\tau_0 = 83.8 \ N/m^2 \ Ans$$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

for trailing edge $x = L$

$$\delta = \frac{5L}{\sqrt{Re_L}} = \frac{5 \times 0.95}{3 \times 10^4}$$

$$= 0.0129 \ m \ Ans$$

Q) for air over a flat plate the velocity u and boundry layer thickness δ could be expressed as $\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$ and $\delta = \frac{4.64x}{\sqrt{Re_x}}$ if the free stream velocity is 2 m/sec. and kinematic viscosity $= 1.5 \times 10^{-5} \ m^2/sec$. $\rho = 1.23 \ kg/m^3$. find the shear stress on the surface of the plate at $x = 1m$.

Soln/ Self M

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}$$

$$\tau_0 = \rho U_\infty^2 \frac{d\theta}{dx}$$

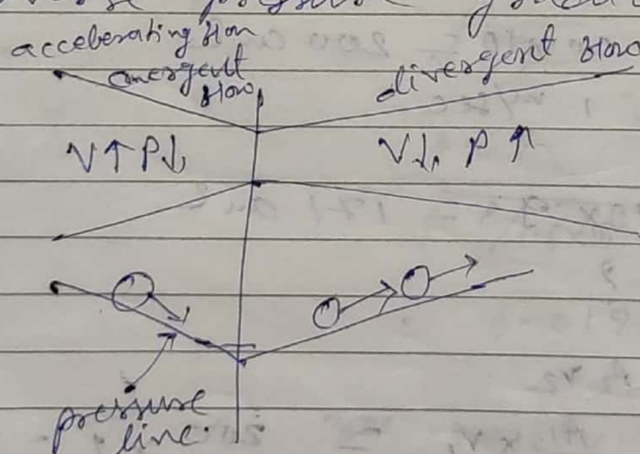
$$\frac{d\theta}{dx} = \frac{d\theta}{dx}$$

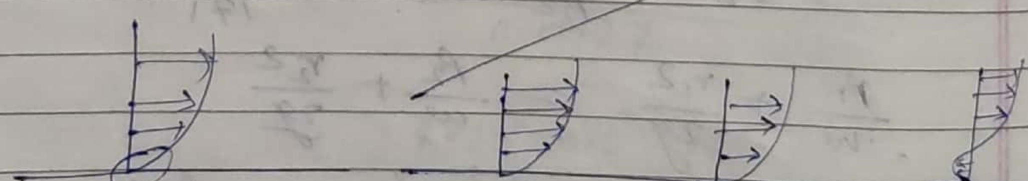
$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy$$

As viscous force are neglected outside the boundry layer so we can apply Bernoulli's eqn.

Boundry layer seperation :→

In case of convergent flow the velocity increases and pressure decreases therefore the fluid slides down the pressure hill. In case of divergent flow the velocity decreases and pressure increases i.e the fluid moves under +ve pressure gradient. And hence it has to climb a pressure hill. If the velocity reduction is more momentum of particle may not support to climb pressure line and hence flow may reverse its direction from boundry and this is known as boundry layer seperation and reversal of flow and this occurs under +ve pressure gradient and these +ve pressure gradient are also known as adverse pressure gradient.





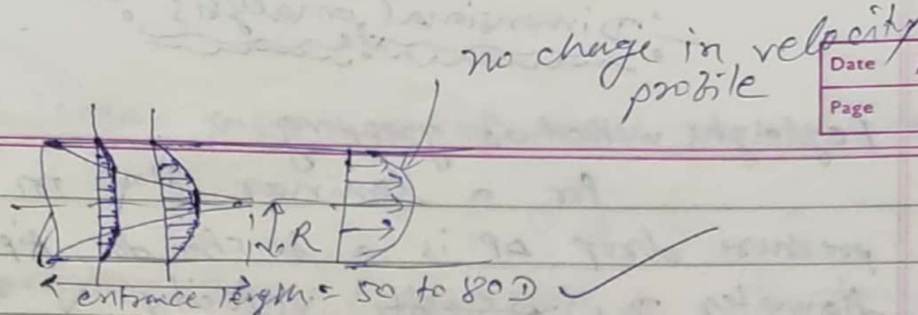
$\tau = \mu \frac{du}{dy}$

$\tau_0 = \mu \frac{du}{dy}\Big|_{y=0}$

$\frac{du}{dy}\Big|_{y=0} = +ve$ — No seperation, Attatched

$\frac{du}{dy}\Big|_{y=0} \to 0$ — about to seperate, seperation point

$\frac{du}{dy}\Big|_{y=0} = -ve$ — Seperated



Note:- The maxm thickness of boundry layer in a pipe is equal to radius of the pipe.

for a fully developed flow the velocity profile does not change.