Signal and system
4th semester (EE)

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Behaviour of Continuous and discrete time LTI system (Linear Time invariant system)

I/p = Input

Step response and Impulse responses.

I/p - o/p behaviour with aperiodic convergent Input

Let us consider R-L series ckt. in the ckt the switch is (S) closed at t = 0, for step response the I/p excitation is $x(t) = Vu(t)$. Applying KVL to the ckt.

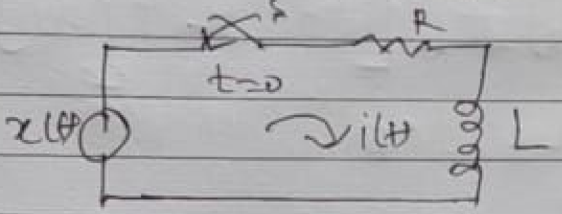


fig. R-L ckt

$$Vu(t) = Ri(t) + L\frac{di(t)}{dt}$$

Taking laplace Transform on both side, we have

$$\frac{V}{s} = RI(s) + L\left[sI(s) - i(0^+)\right]$$

Since the current through Inductor cannot change Instantaneously and since the current before $t = 0$ is zero

$$i(0^+) = 0$$

$$\frac{V}{s} = RI(s) + LsI(s)$$

$$= [R + Ls]\, I(s)$$

$$\therefore I(s) = \frac{V}{s}\frac{1}{(R+Ls)}$$

$$= \frac{V}{s(R+Ls)}$$

By partial fraction

$$= \frac{V}{L}\frac{1}{s\left(s+\frac{R}{L}\right)} = \frac{V}{\cancel{L}}\cdot\frac{\cancel{L}}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$$

$$I(s) = \frac{V}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$$

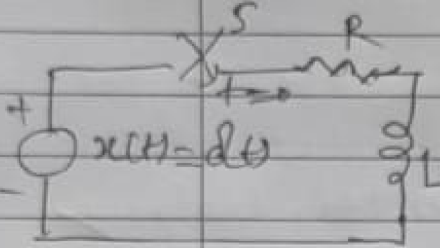
Taking ILT, we get

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]}$$

Impulse response

We know that the impulse response input excitation is $x(t) = \delta(t)$

The differential eqn describing the R-L network would become



$$\delta(t) = R\,i(t) + L\frac{di(t)}{dt}$$

Taking Laplace Transform on both sides, we have

$$1 = R\,I(s) + L\left[s\,I(s) - \underset{0}{\underbrace{I(0^+)}}\right]$$

$$1 = (R + Ls)\,I(s)$$

$$I(s) = \frac{1}{R + Ls}$$

$$= \frac{1}{L}\cdot\frac{1}{\left(s + \frac{R}{L}\right)}$$

Taking ILT on both side.

$$\boxed{i(t) = \frac{1}{L}e^{-\left(\frac{R}{L}\right)t}u(t)}$$

Step and Impulse responses of series R-C ckt

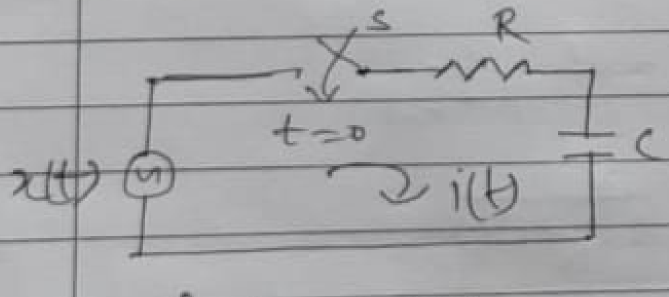


fig: R-C ckt

~~In the series R-C series ckt is shown. let us consider that the switch S is closed~~

~~we know that the for step response the input excitation is $x(t) = V u(t)$.~~

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$$Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = Vu(t)$$

$$Ri(t) + \frac{1}{C}\int_{0}^{t} i(t)\,dt + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt = Vu(t)$$

Talking laplace Transform on both sides, we have

$$RI(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right] + \frac{1}{C}\left\{L[q(0^+)]\right\} = \frac{V}{s}.$$

$$RI(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{q(0^+)}{s}\right] = \frac{V}{s}$$

where $q(0^+)$ is the charge on the capacitor at $t = 0^+$ if the capacitor is initially relaxed $q(0^+) = 0$

$$RI(s) + \frac{1}{Cs}I(s) = \frac{V}{s}$$

$$I(s)\left[R + \frac{1}{Cs}\right] = \frac{V}{s}$$

$$I(s) = \frac{V}{s} \cdot \frac{1}{R + \frac{1}{Cs}}$$

$$= \frac{V}{\cancel{s}} \cdot \frac{C\cancel{s}}{RCs + 1}$$

$$= \frac{VC}{RC\left[s + \left(\frac{1}{RC}\right)\right]}$$

$$I(s) = \frac{V/R}{s + \left(\frac{1}{RC}\right)}$$

Taking ILT on both sides, we get

$$\boxed{i(t) = \frac{V}{R}e^{-\frac{1}{RC}t}.}$$

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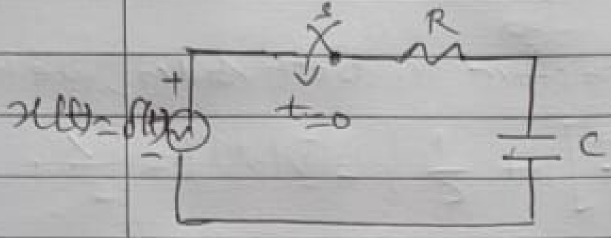
Impulse response

We know that for the impulse response the I/P excitation $x(t) = \delta(t)$ Hence, the ckt



$$\delta(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

$$\delta(t) = Ri(t) + \frac{1}{C}\int_{0}^{t} i(t)\,dt + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt$$

Taking Laplace transform on both sides, we get

$$1 = RI(s) + \frac{1}{C}\left[\frac{I(s)}{s}\right] + \frac{1}{C}\{[q(0^+)]\}$$

$$\Downarrow 0$$

$$1 = \left(R + \frac{1}{Cs}\right) I(s) \qquad [q(0^+) = 0]$$

$\therefore$ if $i = \frac{dq}{dt}$, $\int i(t)\,dt = q$, $\therefore \int i(t)\,dt = q$

$$\therefore I(s) = \frac{1}{\left(R + \frac{1}{Cs}\right)} = \frac{1}{R\left[1 + \left(\frac{1}{RCs}\right)\right]}$$

$$I(s) = \frac{RCs}{R(RCs+1)} = \frac{Cs}{RC\left[s + \left(\frac{1}{RC}\right)\right]}$$

$$= \frac{s}{R\left[s + \left(\frac{1}{RC}\right)\right]}$$

$$I(s) = \frac{1}{R}\left[\frac{s + \left(\frac{1}{RC}\right) - \left(\frac{1}{RC}\right)}{s + \left(\frac{1}{RC}\right)}\right]$$

$$I(s) = \frac{1}{R}\left[1 - \frac{1}{RC} \cdot \frac{1}{\left[s + \left(\frac{1}{RC}\right)\right]}\right]$$

Taking ILT on both sides, we get.

$$\boxed{i(t) = \frac{1}{R}\left[\delta(t) - \frac{1}{RC} \cdot e^{-\frac{1}{RC}t} u(t)\right]}$$

~~Periodic signal which repeats itself at regular interval of time is called periodic signal.~~

~~and a signal which does not repeat~~ at regular interval of time is called a non periodic or aperiodic signal.