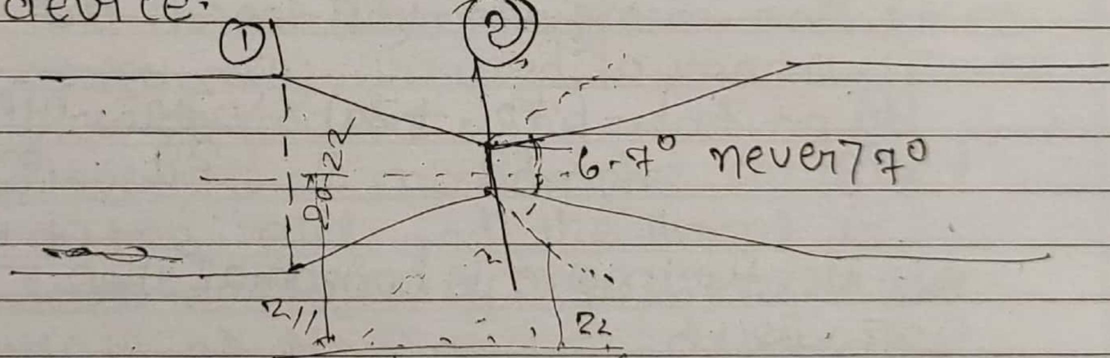
$$\frac{V_2}{V_1} = \sqrt{9} = 3$$

$$\boxed{\frac{V_2}{V_1} = \frac{3}{1}}$$ Ans

Application of Bernoulli's ~~the~~ equation:

① **Venturimetre:→**

venturimetre is a device which is used for finding out the discharge. It is a converging and diverging cross sectional device.



In venturimetre the length of diverging section is greater than that of converging section. Generally the diverging angle is kept less than 7°. In order to avoid flow separation because if we increase diverging angle, the diverging length will decreased and hence flow separation will occur and hence direction of flow is ~~reverse~~ will be reversed. so the diverging angle should be less than converging angle.

It consists of three parts
(i) converging section
(ii) Throat.
(iii) Diverging section

Applying Bernoulli's eqn at the inlet venturimetre i.e, at section (1) and at the throat of venturimetre i.e, at section (2).

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

$$\left(\frac{P_1}{\rho g}+z_1\right)+\frac{v_1^2}{2g}=\left(\frac{P_2}{\rho g}+z_2\right)+\frac{v_2^2}{2g}$$

$$\left(\frac{P_1}{\rho g}+z_1\right)-\left(\frac{P_2}{\rho g}+z_2\right)=\frac{v_2^2-v_1^2}{2g}=h$$ [piezometric head diff]

if venturimetre is horizontal then $z_1=z_2$.

$$\frac{P_1}{\rho g}-\frac{P_2}{\rho g}=\frac{v_2^2-v_1^2}{2g}=h$$ (pressure head diff)

$$\Rightarrow \frac{v_2^2-v_1^2}{2g}=h$$

$$\Rightarrow v_2^2-v_1^2=2gh \quad \text{--- (1)}$$

Now, $Q=a_1v_1=a_2v_2$

$$\Rightarrow v_1=\frac{Q}{a_1} \;\&\; v_2=\frac{Q}{a_2}$$

~~Substituti~~ Substituting v_1 & v_2 in eqn ① we have

$$\left(\frac{Q}{a_2}\right)^2 - \left(\frac{Q}{a_1}\right)^2 = 2gh$$

$$\Rightarrow \frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2} = 2gh$$

$$\Rightarrow Q^2\left[\frac{1}{a_2^2} - \frac{1}{a_1^2}\right] = 2gh$$

$$\Rightarrow Q^2\left[\frac{a_1^2 - a_2^2}{a_1^2 a_2^2}\right] = 2gh$$

$$\Rightarrow Q^2 = \frac{a_1^2 a_2^2}{a_1^2 - a_2^2} \cdot 2gh$$

$$\Rightarrow Q = \sqrt{\frac{a_1^2 \cdot a_2^2}{a_1^2 - a_2^2} \cdot 2gh}$$

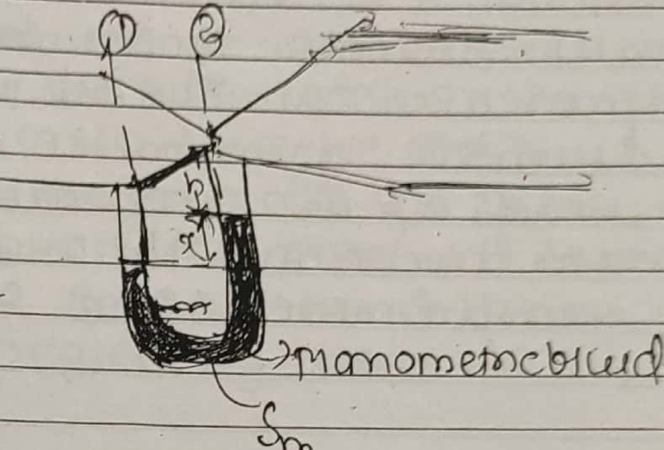
$$\Rightarrow Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\Rightarrow \boxed{Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

This is the required expression for the discharge through venturimetre.

Application

Discharge: Q
Sp. gravity: S →



$q_1 > q_2$

$v_1 < v_2$

$P_1 > P_2$

$$P_1 + \rho g (H + x) - \rho_m g x - \rho g H = P_2$$

$$= \frac{P_1}{\rho g} + (H + x) - \frac{\rho_m \cancel{g} x}{\rho \cancel{g}} - \frac{\cancel{\rho g} H}{\cancel{\rho g}} = \frac{P_2}{\rho g}$$

$$= \frac{P_1}{\rho g} + (H + x) - \frac{\rho_m}{\rho} x - H = \frac{P_2}{\rho g}$$

$$= \frac{P_1}{\rho g} + \cancel{H} + x - \frac{\rho_m}{\rho} x - \cancel{H} = \frac{P_2}{\rho g}$$

$$= \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{\rho_m}{\rho} x - x$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = x\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\boxed{h = x\left(\frac{S_m}{S} - 1\right)}$$

Coefficient of discharge (C_d):- It is defined as the ratio of actual discharge to the theoretical discharge.

$$\boxed{C_d = \frac{Q_{act}}{Q_{th}}}$$

$$\Rightarrow Q_{act} = C_d \cdot Q_{th}$$

$$Q_{act} = C_d \cdot \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

* C_d for venturimetre = 0.95 to 0.99

Q. A horizontal venturimetre with inlet and throat diameter 30cm and 15cm respectivelly is used to measure the flow of water. The reading of differential manometre connected to the inlet and the throat is 20cm of mercury. Determine the rate of flow, i.e, discharge. take $C_d = 0.98$

Sol:

$$Q_{act} = C_d \cdot \frac{\pi}{4} \times (30)^2 \times \frac{\pi}{4} \times$$

Given, $D_1 = 30cm$

$D_2 = 15cm$

$a_1 = \frac{\pi}{4} \times (30)^2 = 706.85 \; cm^2$

$a_2 = \frac{\pi}{4} \times (15)^2 = 176.71 \; cm^2$

$$\boxed{1 l = 10^{-3} m^3}$$

classmate

Date

Page

$h = x\left[\frac{S_m}{S} - 1\right]$

$h = 20\left[13.6 - 1\right]$

$\frac{S_m}{S} = 13.6$

252 cm

$$q_a = \frac{C_d \cdot a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.98 \times 706.85 \times 176.71 \times \sqrt{2 \times 98 \times 252}}{\sqrt{(706.85)^2 - (176.71)^2}}$$

$= 12569.83 \ m^3/\cancel{sec}$

$= 12.569 \ m^3/sec$

$$cd\,\frac{a_1 a_2\sqrt{2g h}}{\sqrt{a_1^2 - a_2^2}}$$

classmate
Date
Page

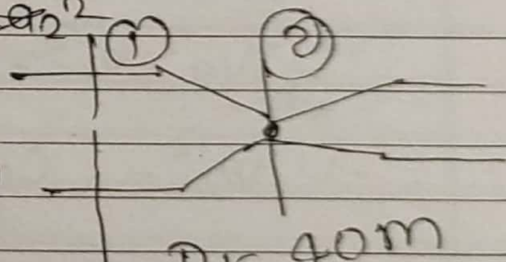
Q3. A venturimetre of 20millmetre throat diametre is installed in a horizontal pipe of 40mill diametre. The fluid flowing is water. If the pressure difference between pipe and throat section is 30kPa, find the velocity in pipe neglecting losses $\frac{P_1 - P_2}{\rho g} = 30000$

Soln

$d_1 = 20$ millimetre

$d_2 = 40$ milldiametre

$$Q_{ct} = \frac{\cancel{cd\, a_1 a_2 \sqrt{2gh}}}{\sqrt{a_1^2 - a_2^2}}$$



$a_1 = \cancel{\frac{\pi}{4}} \times (20)$

$\cancel{\frac{v_2^2 - v_1^2}{2}} =$

$D_1 = 40m$
$D_2 = 20mm$
$\frac{P_1 - P_2}{\rho g} = 30{,}000$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$= \frac{30 \times 10^3}{\rho \cancel{g}} = \frac{v_2^2 - v_1^2}{2\cancel{g}}$$

$$= v_2^2 - v_1^2 = \frac{2 \times 30 \times 10^3}{1000}$$

$v_2^2 + v_1^2 = 60$ — (1)

$$\frac{a_1 a_2 V_{2g}}{\sqrt{a_1^2 - a_2^2}}$$

$$h = x\left[\frac{S_m}{S} - 1\right]$$

classmate
Date
Page

$$a_1 V_1 = a_2 V_2$$

$$\frac{a_1}{a_2} = \frac{V_2}{V_1}$$

$$= \frac{\cancel{\frac{\pi}{4}} \times D_1^2}{\cancel{\frac{\pi}{4}} \times D_2^2} = \frac{V_2}{V_1}$$

$$= \frac{(40)^2}{(20)^2} = \frac{V_2}{V_1}$$

$$= \frac{\cancel{1600}^{4}}{\cancel{400}} = \frac{V_2}{V_1}$$

$$V_2 = 4V_1 \quad \text{— (1)}$$

$$(4V_1)^2 - V_1^2 = 60$$

$$16V_1^2 - V_1^2 = 60$$

$$15V_1^2 = 60$$

$$V_1^2 = \frac{\cancel{60}^{4}}{\cancel{15}}$$

$$\boxed{V_1 = 2\ m/s}$$

$$V_2 = 4V_1$$

$$V_2 = 4 \times 2$$

$$\boxed{V_2 = 8\ m/s}$$

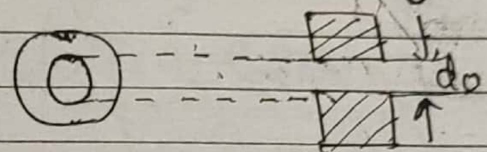
$V_1 = 2$ m/se

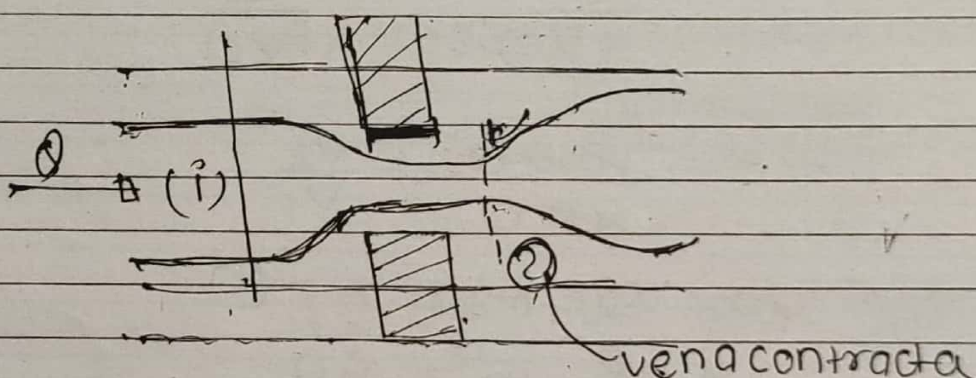
$V_2 = 8$ m/sec

B

Orbice meter: →

It is used bor binding out discharge and is based on the same principle as that of venturimetre and is the cheapest instrument bor binding the discharge.





* coebbicient of contraction (C_c)

$$\boxed{C_c = \frac{a_2}{a_0}}$$

$$\Rightarrow \boxed{a_2 = C_c \cdot a_0}$$

$$a_1 v_1 = a_2 v_2$$

$$a_1 v_1 = C_c \cdot a_0 \cdot v_2$$

$$v_1 = C_c \cdot \frac{a_0}{a_1} \cdot v_2$$

Applying Bernoulli's eq^n between ① and ②.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

for horizontal pipe, $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h \text{ (pressure head difference)}$$

$$\Rightarrow \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h$$

$$\Rightarrow V_2^2 - V_1^2 = 2gh$$

$$\Rightarrow V_2^2 - \left(C_c \frac{a_0}{a_1} \cdot V_2\right)^2 = 2gh$$

$$\Rightarrow V_2^2 \left[1 - \frac{C_c^2 a_0^2}{a_1^2}\right] = 2gh$$

$$\Rightarrow V_2^2 = \frac{2gh}{1 - \frac{C_c^2 a_0^2}{a_1}}$$

$$\Rightarrow \boxed{V_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}}$$

Now,

discharge $Q = a_2 v_2$

$$Q = \frac{C_c \cdot a_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}$$

$$Q = \frac{C_c\, a_0 \sqrt{2gh}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}} \times \frac{\sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{a_0^2}{a_1^2}}}$$

If $C_d = C_c \cdot \frac{\sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1^2}}}$

then

$$Q = \frac{C_d \cdot a_0 \sqrt{2gh}}{\sqrt{1 - \frac{a_0^2}{a_1^2}}}$$

$$Q = \frac{C_d \cdot a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$$\boxed{Q = \frac{C_d\, a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

for oribice meter, the value ob C_d is $\boxed{0.68 \text{ to } 0.76}$

$$C_d = \frac{C_c \cdot \sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - C_c^2 \frac{a_0^2}{a_1}}}$$

in some books

the term $\dfrac{\sqrt{1 - \frac{a_0^2}{a_1^2}}}{\sqrt{1 - \frac{C_c^2 a_0^2}{a_1 2}}}$ is termed

as coebbicent ob velocity

so $\boxed{C_d = C_c \cdot C_v}$

classmate
Date
Page

Q) An orifice metre is being used for measuring flow rate of liquid in a pipeline source a pressure diff of x metre of water column when the flow is Q. If the flow rate is ~~to~~ is double, the pressure head difference in m is

(a) $2x$ (b) $8x$ (c) x^2 (d) $4x$

Sol:

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h \text{ (head)}$$

$$Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

$Q \propto \sqrt{h}$

$Q = K\sqrt{h}$

$Q_1 = K\sqrt{h_1}$ (i)

$Q_2 = K\sqrt{h_2}$ (ii)

$$\frac{Q_1}{Q_2} = \sqrt{\frac{h_1}{h_2}}$$

$$\frac{\cancel{Q}}{2\cancel{Q}} = \sqrt{\frac{x}{h_2}}$$

$$\therefore \frac{1}{2} = \sqrt{\frac{x}{h_2}}$$

squaring

$$\frac{1}{4} = \frac{x}{h_2}$$

$$\boxed{h_2 = 4x}$$

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Date
Page

Q2. An orifice metre with Cd = 0.61 is substituated by venturimetre with Cd = 0.98 in a pipe line carrying oil having same throat diametre as that of orifice metre. For the same flow rate. What will be ratio of pressure drops for venturimetre to that orifice metre.

Sol

$$\cancel{Q = \frac{Cd \cdot a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}} =$$

$(Cd)_v = 0.98 \quad (Cd)_o = 0.61$

$D_t = D_o$

$a_t = a_o$

$a_2 = a_o$

$Q_v = Q_o$

$$\frac{(Cd)_v \cancel{a_1 a_2} \sqrt{2gh_v}}{\cancel{\sqrt{a_1^2 - a_2^2}}} = \frac{(Cd)_o \cancel{a_1 a_o} \sqrt{2gh_o}}{\cancel{\sqrt{a_1^2 - a_o^2}}}$$

$$\cancel{\frac{0.98}{0.61} = \sqrt{2gh_o}}$$

$$\frac{(Cd)_v}{(Cd)_o} = \sqrt{\frac{\cancel{2g}h_o}{\cancel{2g}h_v}}$$

$$\left(\frac{0.98}{0.61}\right)^2 = \frac{h_o}{h_v}$$

$$\frac{h_o}{h_v} = \frac{(0.98)^2}{(0.61)^2}$$

$$\frac{h_o}{h_v} =$$

classmate
Date
Page

$$\frac{h_v}{h_o} = \frac{0.61^2}{0.98^2}$$

$$= \frac{\frac{(P_1-P_2)_v}{\cancel{\rho g}}}{\frac{(P_1-P_2)_o}{\cancel{\rho g}}} = \frac{0.61^2}{0.98^2}$$

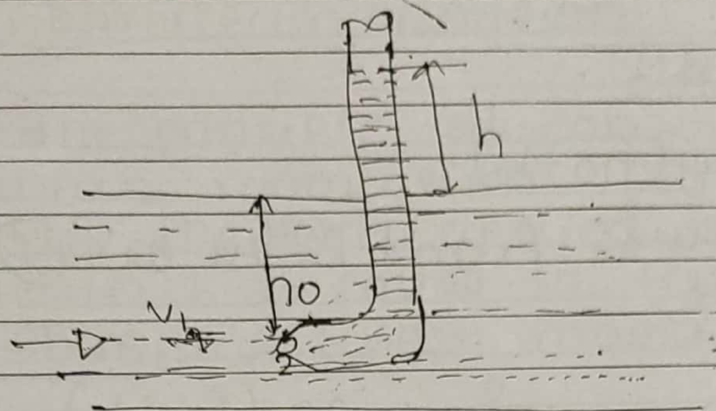
$$= \frac{(P_1-P_2)_v}{(P_1-P_2)_o} = \frac{0.61^2}{0.98^2}$$

~~P~~

~~PEPO~~

* Pitot tube:→ It is based on the principle that if the velocity of flow at any point becomes zero the pressure is increased due to change of K.E into pressure energy.

Pitot tube is used for finding out velocity of flow at any point in a pipe or channel.



consider two points 1 and 2 at the same level in such a way that point 2 is just the inlet of the pitot tube and point 1 is far away from the tube. since at 2 velocity becomes zero so P_2 is increased and the liquid rises up the pitot tube as shown in the figure.

Appling Bernoulli's equation b/w pt ① and ② for horizontal pipe, we have

$$\frac{P_1}{\omega} + \frac{v_1^2}{2g} = \frac{P_2}{\omega} + \frac{v_2^2}{2g}$$

since at 2, velocity = 0

so, $v_2 = 0$ hence the above eqn becomes

$$\underbrace{\frac{P_1}{\omega}}_{\text{Static head}} + \underbrace{\frac{v_1^2}{2g}}_{\text{Kinetic head}} = \underbrace{\frac{P_2}{\omega}}_{\text{Stagnation head}} \quad \text{--- ①}$$

classmate

Date
Page

here,

$P_1 = \rho g h_0$

$P_2 = \rho g (h_0 + h)$

substituting P_1 and P_2 in eqn (1)
we get,

$$\frac{\rho g h_0}{w} + \frac{v_1^2}{2g} = \frac{\rho g (h_0 + h)}{w}$$

Putting $w = \rho g = $ sp. wt.

$$\frac{\cancel{\rho g} h_0}{\cancel{\rho g}} + \frac{v_1^2}{2g} = \frac{\cancel{\rho g}(h_0 + h)}{\cancel{\rho g}}$$

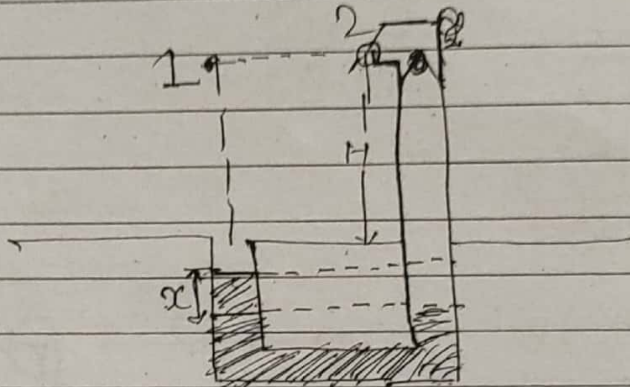
$$\Rightarrow h_0 + \frac{v_1^2}{2g} = h_0 + h$$

$$\Rightarrow \frac{v_1^2}{2g} = h$$

$$\Rightarrow v_1^2 = 2gh$$

$$\boxed{v_1 = \sqrt{2gh}}$$

velocity in pipes :



Applying Bernoulli's equation at 1 and 2

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

If $z_1 = z_2$ if $V_2 = 0$ then

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} \quad \text{—— (ii)}$$

from balancing of fluid column technique

$$P_1 + \rho g H + \rho_m g x - \rho g (H+x) = P_2$$

dividing by ρg

$$\frac{P_1}{\rho g} + \frac{\cancel{\rho g} H}{\cancel{\rho g}} + \frac{\rho_m \cancel{g} x}{\rho \cancel{g}} - \frac{\cancel{\rho g}(H+x)}{\cancel{\rho g}} = \frac{P_2}{\rho g}$$

$$\frac{P_1}{w} + \cancel{H} + \frac{\rho_m}{\rho} x - \cancel{H} - x = \frac{P_2}{w}$$

CLASSMATE

Date

Page

$$\Rightarrow \frac{P_1}{\omega} + \frac{S_m}{S}x - x = \frac{P_2}{\omega}$$

$$\Rightarrow \frac{P_1}{\omega} = \frac{P_2}{\omega} + x\left(1 - \frac{S_m}{S}\right)$$

Substituting ~~the putting the~~ $\frac{P_1}{\omega}$ in eq^n (i)

$$\cancel{\frac{P_2}{\omega}} + x\left(1 - \frac{S_m}{S}\right) + \frac{V_1^2}{2g} = \cancel{\frac{P_2}{\omega}}$$

$$\frac{V_1^2}{2g} = x\left(\frac{S_m}{S} - 1\right)$$

$$\Rightarrow \boxed{V_1 = \sqrt{2gx\left(\frac{S_m}{S} - 1\right)}}$$

Q. A pitot tube is used to measure velocity of water using differential gauge which contains manometric fluid of specific gravity 1.4. ~~find~~ .9b the velocity of water is 1.2 m/sec find manometric fluid deflection.

Soln:

$$V_1 = \sqrt{2gx\left(\frac{S_m}{S} - 1\right)}$$

$$(1.2)^2 = \cancel{\sqrt{\ }}\ 2gx\left(\frac{1.4}{1} - 1\right)$$

$$2gx(0.4) = (1.2)^2$$

$$x = \frac{(1.2)^2}{2g \times 0.4} = 0.183\ m$$