

"Analogous system"

"Physical system"

Mathematical Model of physical system

Modelling of mechanical system:

① Translational motion

② Rotational motion

The Motion of the body during translational motion is along a straight line or curve path. Whereas during rotation motion, the motion of the body is about its own axis.

Translation motion: There are three elements which are fundamentally involved in the analysis of translation motion. These are

(a) Mass (b) Spring (c) Damper

(a) Mass: In the physical model of a mass, it is assumed that its mass is concentrated at the centre of the mass.

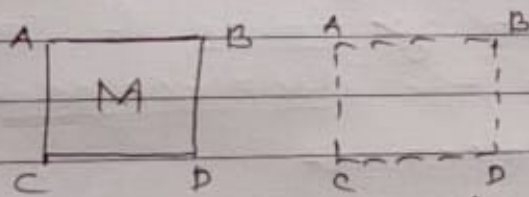


fig ① movement of a block of mass.

Fig ① shows that when a block ABCD of mass  $M$  is moved all its particles undergo an equal displacement.

Therefore, mass has only one displacement.

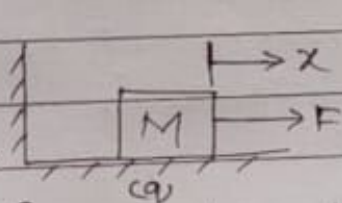
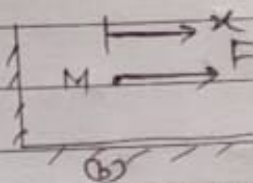


fig ② Displacement of mass



equivalent displacement of mass.

fig ② only one displacement is enough for mass.

$x \rightarrow$  can be put any where if the mass as reference

$x \rightarrow$  represent the displacement w.r.t reference

The relation between force (F) and mass (M) is given by

$$F = Ma$$

M  $\rightarrow$  MASS of the body

a  $\rightarrow$  is the ACCEL<sup>n</sup> of the body

u  $\rightarrow$  MASS of the velocity

x  $\rightarrow$  is the displacement of the mass.

$$M \frac{dv}{dt}$$

$$M \frac{d}{dt} \left( \frac{dx}{dt} \right)$$

$$M \frac{d^2x}{dt^2}$$

$$v = \frac{dx}{dt}$$

$$= M \frac{du}{dt} = M \frac{d}{dt} \left( \frac{dx}{dt} \right)$$
  
$$= M \frac{d^2x}{dt^2}$$

[According to Newton's 2<sup>nd</sup> law of motion inertia force will be equal to the product of mass M and acceleration a.]

(b) Spring: A spring stores the potential energy. the restoring force of spring is proportional to the displacement f & displacement

When a force is applied to the spring (in fig) then the spring force (restoring force) or reaction force is produced which oppose the motion of spring and it is proportional to the displacement (Hook's law) is given by

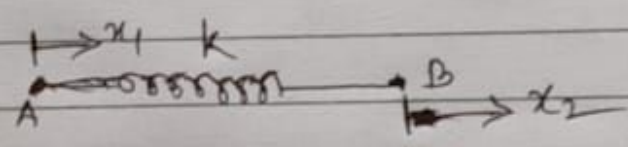


fig: Spring k

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We know that

$$f \propto x$$

$$f = kx \quad \text{--- (1)}$$

$x$  is displacement

$k$  is spring.

Let  $x_1$  is the displacement of end A.

$x_2$  " " " " " B.

If the force is applied at end A.

$$x = x_1 - x_2 \quad \therefore f = k(x_1 - x_2) \quad \text{--- (2)}$$

If the force is applied at end B.

$$x = x_2 - x_1 \quad \therefore f = k(x_2 - x_1) \quad \text{--- (3)}$$

(C) Damper: It is denoted by D.

one of best example for damper are dashpot or shock absorber, consist of piston and cylinder.

~~are the another example.~~

~~motion is opposed by friction.~~

~~frictional force are three types~~

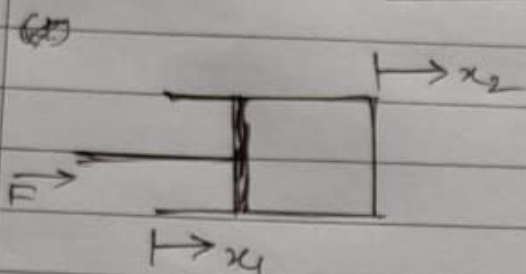


fig: Dashpot

Force opposes the applied force

When a force is applied

to the damper as shown

in fig. Then the damping

force for viscous friction

is proportional to the

difference in velocity

of two ends of the damper

and is given by.

$f \propto$  velocity

$$f = D \frac{dx}{dt} = D \frac{dx}{dt}$$

$$= D \frac{d}{dt} (x_1 - x_2)$$

$x_1$  &  $x_2$  are displacement

here  $x_1$  &  $x_2$  are distance

in velocity.

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