Long division method

Determine the 1st few terms of the sequence $f(k)$ when

$$f(z) = \frac{10z}{(z-1)(z-2)} \quad \text{using long division method}$$

Soln: given $F(z) = \dfrac{10z}{(z-1)(z-2)}$ — ①

$$F(z) = \frac{10z}{z^2-3z+2}$$ नूमीरेटर, ओकेमिटे

Here order of numerator is less than the order of denominator. डिनोमिनेटर

Divide numerator and denominator by z^2

$$F(z) = \frac{10z/z^2}{1-3z^{-1}+2z^{-2}}$$

डिवीजन प्रोसेडर

$$F(z) = \frac{10z^{-1}}{1-3z^{-1}+2z^{-2}} \quad — ②$$

Division procedure:

$$
\begin{array}{r|l}
 & 10z^{-1}+30z^{-2}+70z^{-3}+150z^{-4} \\
1-3z^{-1}+2z^{-2}) & \cancel{10z^{-1}} \\
 & \cancel{10z^{-1}}-30z^{-2}+20z^{-3} \\
 & -\quad + \quad - \\
\hline
 & \cancel{30z^{-2}}-20z^{-3} \\
 & \cancel{30z^{-2}}-90z^{-3}+60z^{-4} \\
 & -\quad + \quad - \\
\hline
 & \cancel{70z^{-3}}-60z^{-4} \\
 & \cancel{70z^{-3}}-210z^{-4}+140z^{-5} \\
 & -\quad + \quad - \\
\hline
 & \cancel{150z^{-4}}-140z^{-5} \\
 & \cancel{150z^{-4}}-450z^{-5}+300z^{-6} \\
 & -\quad + \quad - \\
\hline
 & 310z^{-5}-300z^{-6}
\end{array}
$$

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So eqn ② can be written as

$$\boxed{F(z) = 10z^{-1}+30z^{-2}+70z^{-3}+150z^{-4}+\cdots} \quad — ③$$

M - 525
T - 225
W - 525
Th - 225
Fri - 450
Sat - 375

we know that

$$Z[f(kT)] = F(z) = \sum_{k=0}^{\infty} f(kT) z^{-k}$$

$$= f(0) z^{0} + f(T) z^{-1} + f(2T) z^{-2} + f(3T) z^{-3} + f(4T) z^{-4} \quad \text{— (4)}$$

Comparing eqn (3) & (4)

$f(0) = 0$

$f(T) = 10$

$f(2T) = 30$

$f(3T) = 70$

$f(4T) = 150$

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(3) Find the inverse Z-transform of the given function

$$F(z) = \frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$$ using partial fraction method & Residue Method.

Sol: ~~Partial fraction method~~ :

given function

$$F(z) = \frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$$

$$\frac{F(z)}{z} = \frac{(1-e^{-\alpha T})}{(z-1)(z-e^{-\alpha T})} \quad -(1)$$

let $$\frac{1-e^{-\alpha T}}{(z-1)(z-e^{-\alpha T})} = \frac{A_1}{(z-1)} + \frac{A_2}{(z-e^{-\alpha T})} \quad -(2)$$

A_1 & A_2 are constt

calculate the value of A_1 & A_2

$$A_1 = \cancel{(z-1)} \frac{(1-e^{-\alpha T})}{\cancel{(z-1)}(z-e^{-\alpha T})}\Bigg|_{z=1}$$

$$= \frac{\cancel{(1-e^{-\alpha T})}}{\cancel{(1-e^{-\alpha T})}} = 1$$

$$\boxed{A_1 = 1}$$

Now, $$A_2 = \cancel{(z-e^{-\alpha T})} \frac{(1-e^{-\alpha T})}{(z-1)\cancel{(z-e^{-\alpha T})}}\Bigg|_{z=e^{-\alpha T}}$$

$$= \frac{1-e^{-\alpha T}}{(e^{-\alpha T}-1)}$$

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$$= \frac{\cancel{(1-e^{-\alpha T})}}{-\cancel{(1-e^{-\alpha T})}} = -1$$

$$\boxed{A_2 = -1}$$

Putting the value of A_1 & A_2 in eqn (2)

$$\frac{F(z)}{z} = \frac{(1-e^{-\alpha T})}{(z-1)(z-e^{-\alpha T})} = \frac{1}{(z-1)} + \frac{-1}{z-e^{-\alpha T}}$$

$$\cancel{\frac{F(z)}{z} = \frac{z}{z-1}}$$

$$\frac{F(z)}{z} = \frac{1}{z-1} + \frac{-1}{(z-e^{-\alpha T})}$$

$$\cancel{F(z) = \frac{z}{z-1}}$$

$$F(z) = \frac{z}{z-1} - \frac{z}{z-e^{-\alpha T}}$$

Taking inverse z-transform

$$f(kT) = (1)^k - e^{-\alpha kT}$$

$$\boxed{f(kT) = 1^k - e^{-\alpha kT}} \quad -(3)$$

when $k=0$ $f(0) = 1^0 - e^{-0} = 1-1 = 0$ $\therefore f(0) =$

when $k=1$, $f(T) = 1 - e^{-\alpha T}$

when $k=2$, $f(2T) = 1 - e^{-2\alpha T}$

when $k=3$, $f(3T) = 1 - e^{-3\alpha T}$

= = – – – – – – – –

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Residue method:

$$F(z) = \frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}$$

$f(kT) = \Sigma$ Residue of $F(z)\cdot z^{k-1}$ at $z=1$

$+ \Sigma$ Residue of $F(z)\cdot z^{k-1}$ at $z = e^{-\alpha T}$

$+ \Sigma$ Residue of $F(z)\cdot z^{k-1}$ at – – – –

Solve by $(z-1)F(z)|_{z=1}$ or $(z-1)\cdot\frac{(1-e^{-\alpha T})z}{(z-1)(z-e^{-\alpha T})}\cdot z^{k-1}\Big|_{z=1}$

$$\Rightarrow \cancel{(z-1)}\frac{(1-e^{-\alpha T})z}{\cancel{(z-1)}(z-e^{-\alpha T})}\cdot z^{k-1}\Big|_{z=1}$$

$$+\cancel{(z-e^{-\alpha T})}\frac{(1-e^{-\alpha T})z}{(z-1)\cancel{(z-e^{-\alpha T})}}\cdot z^{k-1}\Big|_{z=e^{-\alpha T}}$$

$$\Rightarrow \frac{\cancel{(1-e^{-\alpha T})}\cdot 1}{\cancel{(1-e^{-\alpha T})}}1^{k}$$

$z^1 \cdot z^{k-1}$
z^{1+k-1}
$z^k = e^{-\alpha T k}$

$$+\frac{(1-e^{-\alpha T})(e^{-\alpha T})}{(e^{-\alpha T}-1)}(e^{-\alpha T})^{k}$$

$$\Rightarrow \frac{1}{1}^{k}+\frac{\cancel{(1-e^{-\alpha T})}}{-\cancel{(1-e^{-\alpha T})}}(e^{-\alpha T})^{k}$$

$$\Rightarrow 1^{k}-e^{-\alpha kT}$$

$$\boxed{f(kT)=1^{k}-e^{-\alpha kT}}$$

Such eqn (5) is similar to eqn (3).

$z\cdot z^{k-1}$
$z^1 z^{k-1}$
$\Rightarrow z^k$
$\Rightarrow 1^k$
$z=1$

$z\cdot z^{k-1}$
z^k
$(e^{-\alpha T})^k$
$e^{-\alpha kT}$
$z=e^{-\alpha T}$

when $k=0$, $f(0)=1^0-e^{-0}=1-1=0$
$f(0)=0$
when $k=1$, $f(T)=1-e^{-\alpha T}$
when $k=2$ $f(2T)=1-e^{-2\alpha T}$
when $k=3$ $f(2T)=1-e^{-3\alpha T}$
— — — — — —

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