By: Er. Pranav Kumar

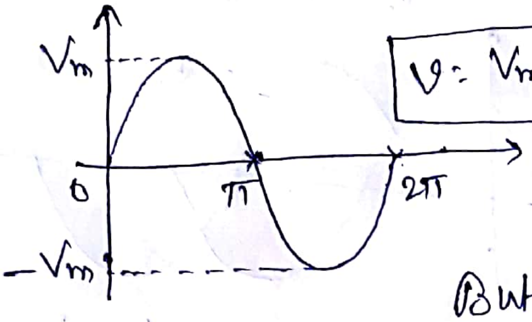
AC steady state (or) circuit Analysis :-

$S^3A \rightarrow$ steady state sinusoidal Analysis.

↳ Sine

Cosine

(a) Radian:



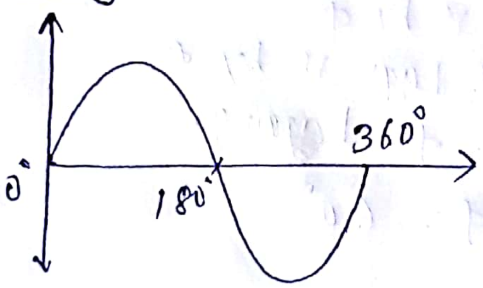
$\boxed{v = V_m \sin \omega t}$

Here $V_m \rightarrow$ Amplitude (voltage)

$\omega =$ Angular velocity (rad/sec)

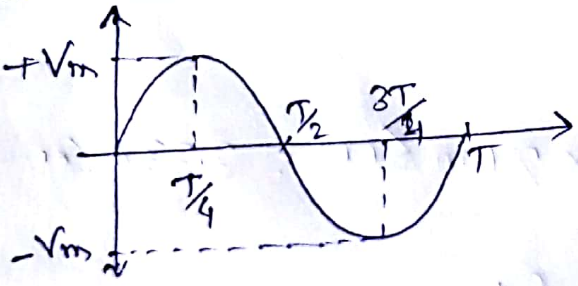
But $f = \frac{1}{T}$ $\left(\omega = \frac{2\pi}{T}\right)$

(b) Degree:



$\boxed{v = V_m \sin \theta}$

(c) Time.



$v = V_m \sin\left(\frac{2\pi}{T} t\right)$

INDIA

$f = 50Hz$ $\quad T = \frac{1}{50}$ sec $= 20$ mSec.

$\omega = 2\pi(50)$

$= 100\pi$ or 314 (rad/sec)

$1T = 2\pi = 360° = 20$ m Sec.

↓ radian ↓ Degree ↓ time

Standard sinusoidal

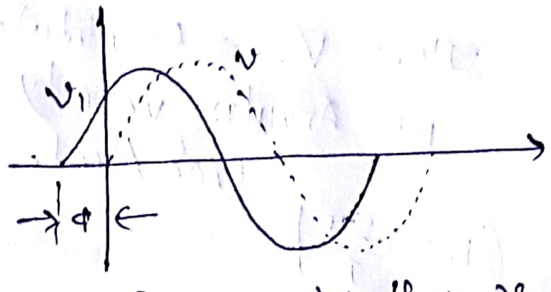
$V_n = V_m \sin(\omega t \pm \phi)$

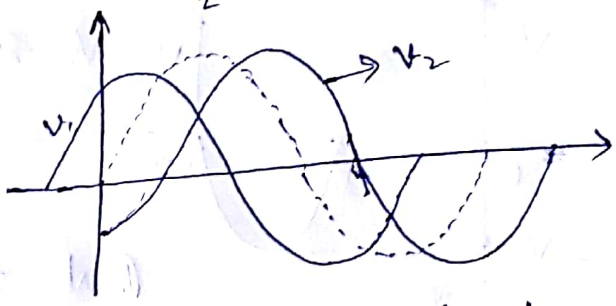
$\phi \rightarrow$ phase shift (deg)

$\downarrow$

Time shifting.

eg:- $V_1 = V_m \sin(\omega t + \phi)$ $\quad$ $V_2 = V_m \sin(\omega t - \phi)$





v_1 Comes early then v by ϕ.

V_1 = Leads v by $\phi°$

$+\phi$ - Leading

$v_2 \rightarrow$ Comes Late to v by ϕ

v_2 Lags v by ϕ

$-\phi$ - Lagging.

India

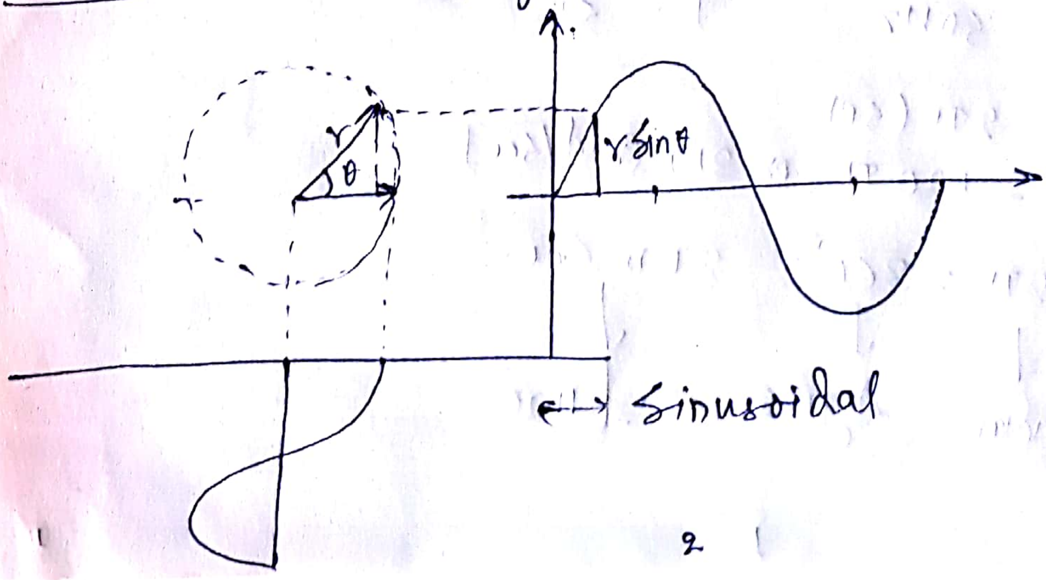
$f = 50Hz$ $\quad$ say $\phi = 60°$

$360° \rightarrow 20 mSec$

$60° \rightarrow t_{shift}$

$$t_{shift} = \frac{60}{360°} \times 20 mSec = 3.33 mSec.$$

PHASOR $\quad$ Represent sinusoidal

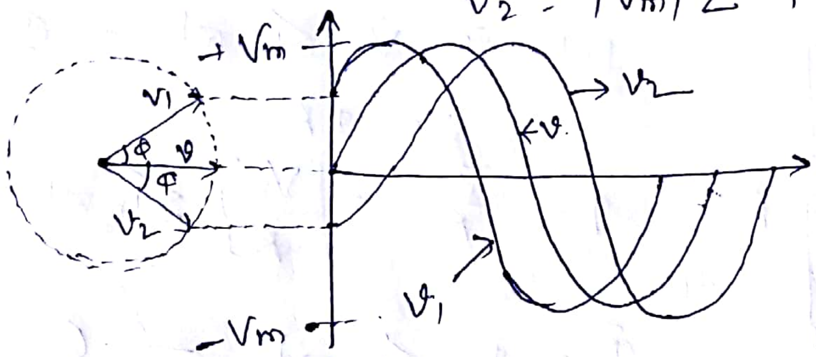


$\leftrightarrow$ Sinusoidal

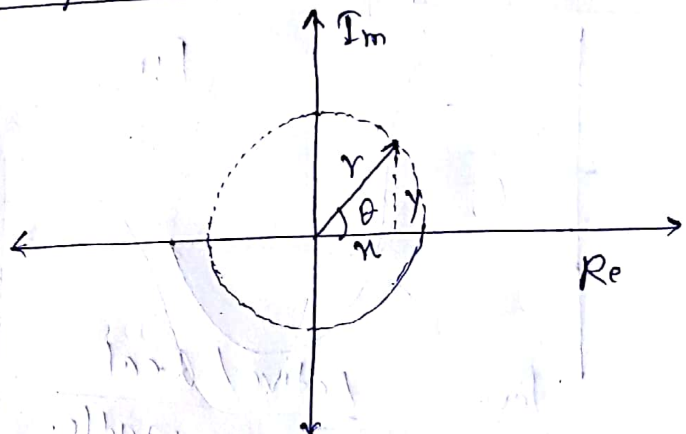
By: Er. Pranav Kumar

$V = |V_m| \angle 0^\circ$ $\quad$ $v_1 = |V_m| \angle +\phi^\circ$

$v_2 = |V_m| \angle -\phi^\circ$



S-plane



$\boxed{P}$ (a) Rectangular $= (x + jy)$

(b) Polar $\rightarrow r \angle \theta$

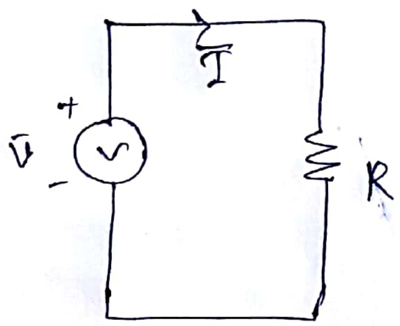
(c) Euler's $\rightarrow r e^{j\theta}$

j is a powerful operator in (E.E) Electrical Engineering

$\boxed{R}$ Rectangular	$\boxed{P}$ Polar
$0 + j0$	$0 \angle 0$
$1 + j0$	$1 \angle 0$
$0 + j1$	$1 \angle 90^\circ$
$1 + j1$	$\sqrt{2} \angle 45^\circ$
$0 - j1$	$1 \angle -90^\circ$

* Phasor Relationship b/w voltage and current in passive elements

ⓐ Resistor



Let $\bar{v} = V_m \sin \omega t$ —— ①

$\because$ $v = IR$

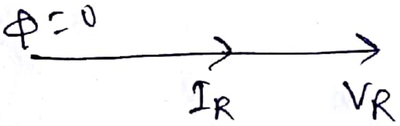
$I = \frac{\bar{v}}{R} = \left[\frac{V_m}{R}\right] \sin \omega t$

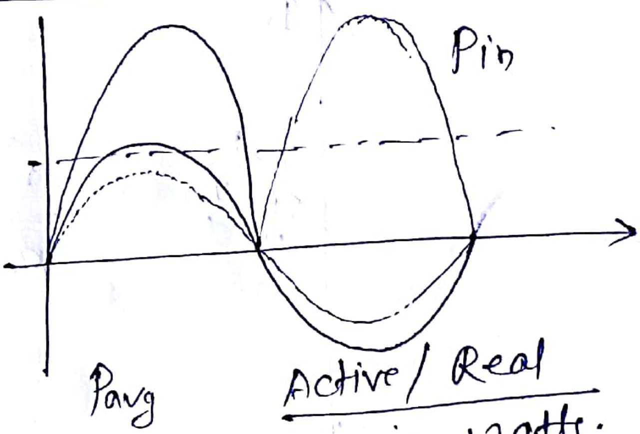
$\bar{I} = I_m \sin \omega t$ —— ②

$$\boxed{\bar{V} = \bar{I} \cdot R}$$

Phasor diagram

$\phi = 0$





Active / Real power in watts.

Power factor :-

$\cos \phi = \cos 0^\circ = 1$ (UPF) Unity power factor.

$$P_{avg} = \frac{1}{T}\int_0^T V(t) \cdot i(t) \, dt = \frac{1}{T}\int_0^T V_m I_m \sin^2 \omega t \, d(\omega t)$$

$$= \frac{V_m I_m}{T}\int_0^T \left[\frac{1 - \cos 2\omega t}{2}\right] d(\omega t)$$

$$P_{avg} = \frac{V_m I_m}{2} \left[\frac{1}{T}\int_0^T d\omega t - \frac{1}{T}\int_0^T \cos 2\omega t \, d(\omega t)\right]$$

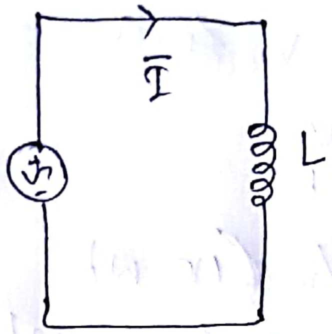
$$P_{avg} = \frac{V_m I_m}{2}\left[1 - 0\right] = \frac{V_m I_m}{2}$$

**

$$\boxed{P_{avg} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms} \text{ (watts)}}$$

By: Er. Pranav Kumar

Inductor:-



Let $\bar{I} = I_m \sin wt$ — ①

$$\boxed{\because \; v = L\frac{dI}{dt}}$$

$\therefore \; v = L \cdot \frac{d}{dt}(I_m \sin wt)$

$= L I_m w \cos wt$

$= wL I_m \sin(90 + wt)$

$\therefore \; \bar{v} = wL I_m \sin(wt + 90^\circ)$ — ②

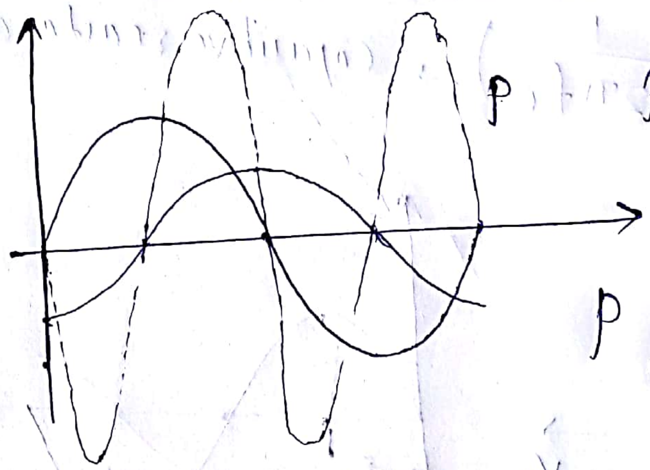
$\bar{v} = wL \underbrace{I_m \sin wt}_{\bar{I}} (j)$

$$\boxed{\bar{v} = j wL \bar{I}}$$

$$\therefore \boxed{\bar{v} = j X_L \bar{I}}$$

$X_L = wL = 2\pi f L$

↳ Inductive Reactance (Ω)

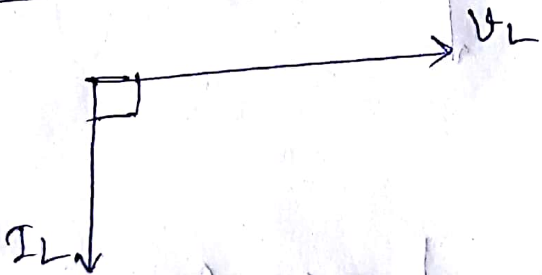


$P_{av} = 0$ zero.

If I & V freq is f then

$P_{freq} = 2f$ (power freq)

Phasor diagram

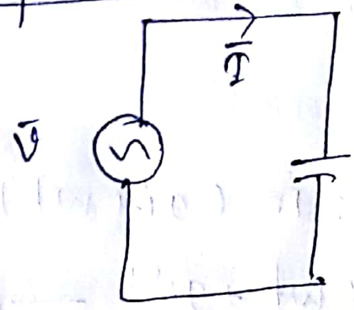


Power factor

$$\boxed{\cos\phi = \cos 90^\circ = 0}$$

(ZPF) (zero power factor)

Capacitor :-



Let $\bar{v} = V_m \sin\omega t$ —— (I)

$$I = C\frac{dv}{dt}$$

$\therefore I = C\omega V_m \cos \omega t$

$= \omega C V_m \sin(90^\circ + \omega t)$

$\therefore I = \omega C V_m \sin(\omega t + 90^\circ)$ —— (II)

$\bar{I} = \omega C \underline{V_m \sin\omega t}\,(j)$

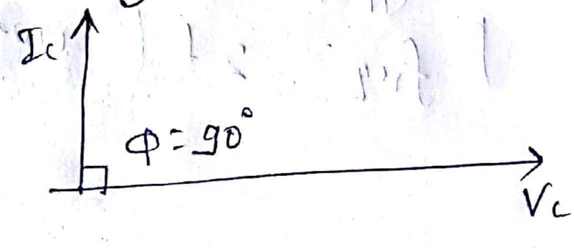
$$\boxed{\bar{I} = j\omega C\,\bar{v}}$$

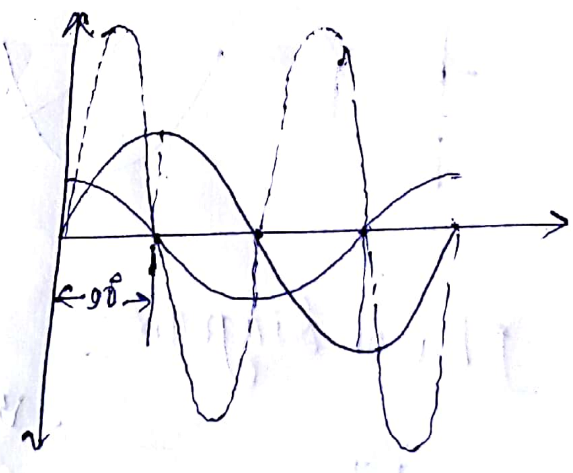
$$\therefore \bar{v} = \frac{\bar{I}}{j\omega C}$$

$$\boxed{\bar{v} = -j X_c \bar{I}}$$

Here $X_c = \frac{1}{\omega C} = \left(\frac{1}{2\pi f C}\right) \rightarrow$ Capacitive reactance (Ω)

Phasor Diagram





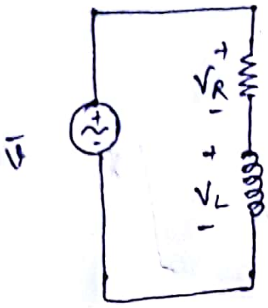
$$\boxed{P_{avg} = 0}$$

Power factor :

$$\boxed{\cos\phi = \cos 90^\circ = 0}$$

ZPF (zero power factor)

(d) **Series R–L** By:- Er. Pranav Kumar



$R \rightarrow R\,\Omega$

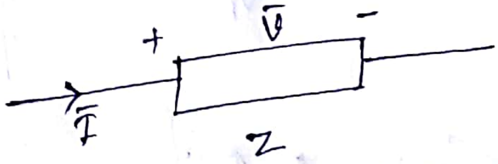
$L \rightarrow +jX_L\,\Omega$

$$\bar{V} = \bar{V}_R + \bar{V}_L$$

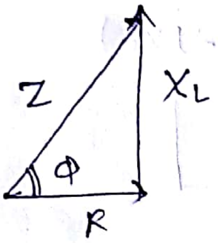
$$= \bar{I}\,(R + jX_L)$$

$$\boxed{\bar{V} = \bar{I} Z}$$

Here $Z = R + jX_L$
↓
Impedance (Ω)



Impedance Δ^{le}



$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

↳ Impedance angle.

Power factor (p.f)

$$\boxed{\cos\phi = \frac{R}{|Z|}}$$

$$\sin\phi = \frac{X_L}{|Z|}$$

$$Z = R + jX_L$$

$$I^2 Z = I^2 R + j I^2 X_L$$

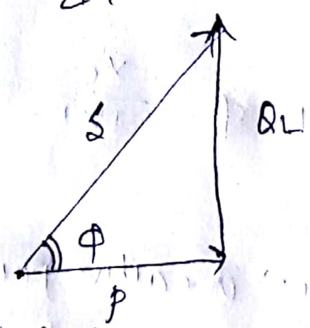
$$\underbrace{VI}_{S} = \underbrace{I^2 R}_{P} + j\,\underbrace{I^2 X_L}_{Q_L}$$

$Q_L \rightarrow$ Reactive power (VAR's)

$I^2 R \rightarrow P \rightarrow$ Real power / Active / True power (watts)

$S \rightarrow$ Total / Apperent power (VA's)

Power Δ^{le}



$$|S| = \sqrt{P^2 + Q_L^2}$$

$$\phi = \tan^{-1}\left(\frac{Q_L}{P}\right)$$

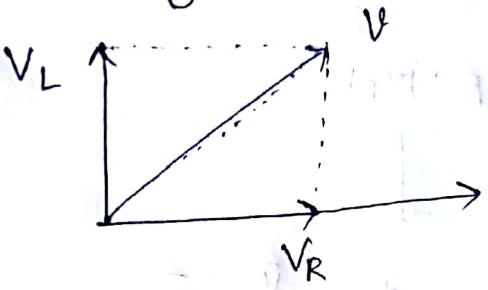
Power factor

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = VI\cos\phi \text{ watts}$$

$$\sin\phi = \frac{Q_L}{S} \Rightarrow Q_L = S\sin\phi = VI\sin\phi \text{ VAR's}$$

V, I r.m.s value.

Phasor diagram :-

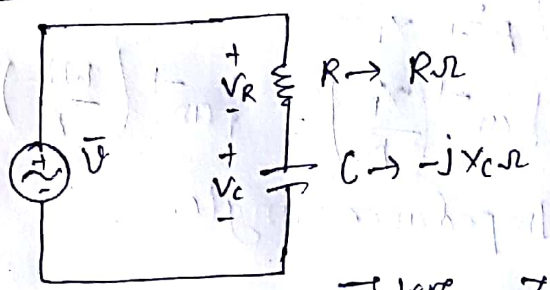


I lags 'V' by $\phi < 90°$

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{V_m \sin \omega t}{R + jX_L}$$

$$\boxed{\bar{I} = \frac{V_m}{|Z|} \sin(\omega t - \phi)}$$ **

Series R-C :



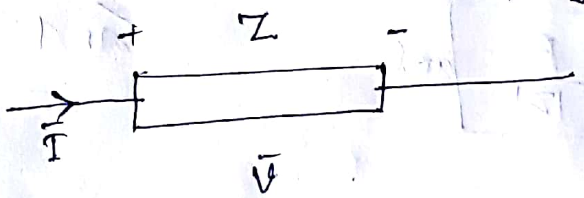
$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$\bar{V} = I(R - jX_C)$$

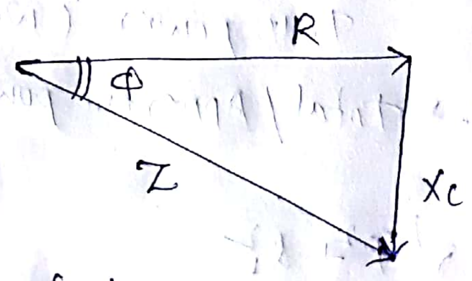
$$\boxed{\bar{V} = \bar{I} Z}$$

Here $Z = R - jX_C$

↳ Impedance.



Impedance Δ^{le} :-



$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

↳ Impedance angle.

Power factor

$$\cos\phi = \frac{R}{|Z|} =$$

$$\sin\phi = \frac{X_C}{|Z|}$$

$$I^2 Z = I^2 R - jX_C I^2$$

$$I \cdot V = I^2 R - jX_C I^2$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

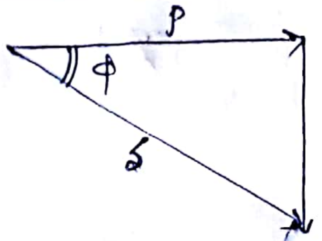
$$S = P - j\,Q_C$$

$Q_C \to$ Realtive power (VAR's)

$P \to$ Real power / Active power

$S \to$ Tolat / Apperent power (VA's)

Power Δ^{le}



$$|S| = \sqrt{P^2 + Q_c^2}$$

$$\phi = \tan^{-1}\left(\frac{Q_c}{P}\right)$$

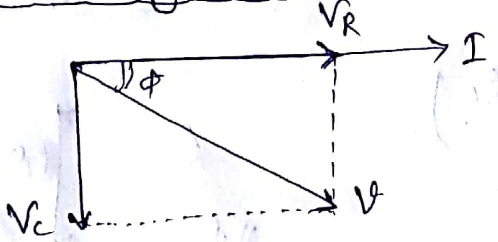
Power factor:-

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = VI\cos\phi \text{ watts}$$

$$\sin\phi = \frac{Q_c}{S} \Rightarrow Q_c = S\sin\phi = VI\sin\phi \text{ VAR's}$$

V, I r.m.s value.

Phasor diagram:-



'I' Leads 'V' by $\phi < 90°$

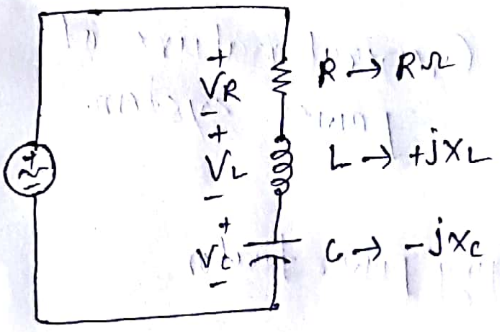
$$\bar{I} = \frac{\bar{V}}{R - jX_c} = \frac{V_m}{|Z|}\sin(\omega t + \phi)$$

Load point of view:-

$+Q_L$ = (Absorbing VAR's Lagging)

$-Q_c$ = (Generating VAR's Leading)

* Series R-L-C



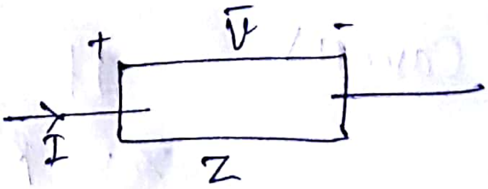
$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_c$$

$$\bar{V} = \bar{I}R - jX_c\bar{I} + jX_L\bar{I}$$

$$\bar{V} = \bar{I}\left[R + j(X_L - X_c)\right]$$

$$\boxed{\bar{V} = \bar{I}Z}$$

Here $Z = R + j\underbrace{(X_L - X_c)}_{X_{net}}$



$$|Z| = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\phi = \tan^{-1}\left[\frac{X_L - X_c}{R}\right]$$

↳ Net impedance angle.

Power factor:-

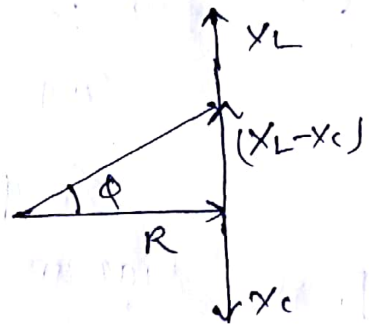
$$\cos\phi = \frac{R}{|Z|} \quad \& \quad \sin\phi = \frac{X_{net}}{|Z|} = \frac{(X_L - X_C)}{|Z|}$$

Also

$$I^2 Z = I^2 R + j\,(I^2 X_L - I^2 X_C)$$

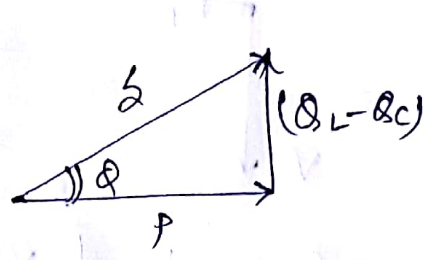
$$V \cdot I = I^2 R + j\,(I^2 X_L - I^2 X_C)$$

$$\boxed{S = P + j\,\underbrace{(Q_L - Q_C)}_{Q_{net}}}$$



$$S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\therefore \phi = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right)$$



Power factor

$$\cos\phi = \frac{P}{S} \quad \therefore \quad P = S\cos\phi = VI\cos\phi \text{ watts.}$$

$$\sin\phi = \frac{Q_{net}}{S} \Rightarrow Q_{net} = S\sin\phi = VI\sin\phi \text{ (VAR's)}$$

Case (I)

If $X_L > X_C$

$Z = R + jX_{net}$

↳ R-L circuit

General nature of power system.

I Lags V by $\phi < 90°$ Lagging power factor

Case (II)

If $X_L < X_C$

$Z = R - jX_{net}$

↳ R-C (circuit)

I Leads V by $\phi < 90°$

Ferranti effect

Case III :- If $X_L = X_C$

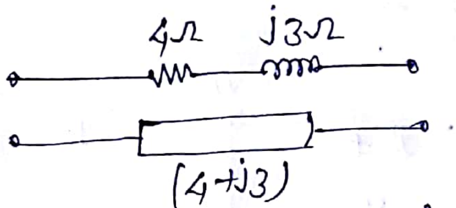
$Z = R \rightarrow$ Purely resistive.

I is in phase with 'V'. Power factor = 1 (UPF)

Resonance.

$$\boxed{\begin{aligned} Z &= R \pm jX \\ \hookrightarrow Z &= R + jX_L \\ Z &= R - jX_C \end{aligned}}$$

eg:-



$Y = \frac{1}{Z}$ (Admittance ℧)

$$Y = \frac{1}{4+j3} \times \frac{4-j3}{4-j3} = \frac{4-j3}{25} = (0.16 - j0.12) ℧.$$

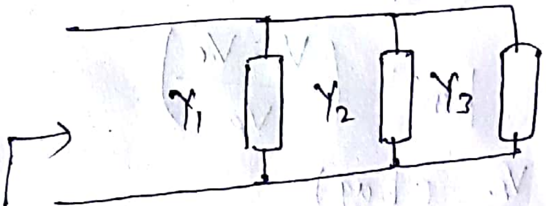
$$\boxed{Y = G \pm jB} \rightarrow Y = G - jB_L$$
$$\rightarrow Y = G + jB_C$$

where $B_L \propto \frac{1}{X_L} \propto \frac{1}{\omega L} \propto \frac{1}{2\pi f L}$ (℧)

$\hookrightarrow$ Inductive susceptance.

$B_C \propto \frac{1}{X_C} \propto \frac{1}{1/\omega C} \propto \omega C \propto 2\pi f C$

$\hookrightarrow$ Capacitive susceptance



$$\boxed{Y_T = Y_1 + Y_2 + Y_3}$$

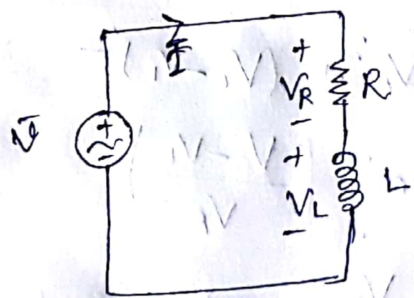
* Phasor diagram :-

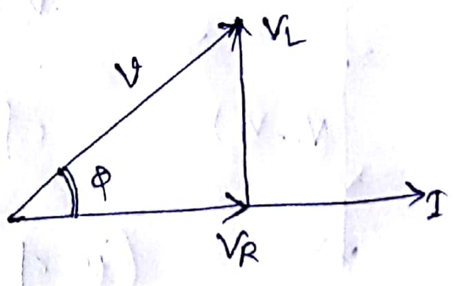
Series circuit. (I, ref.)

(a) Series RL

$V_R = IR\angle 0°$

$V_L = jX_L I = IX_L \angle +90°$



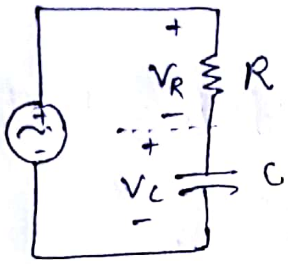


$$V = \sqrt{V_R^2 + V_L^2}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

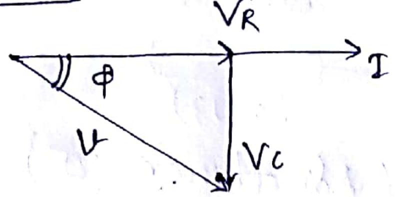
$\cos\phi = \frac{V_R}{V}$ (lagging)

(b) Series R-C :-



$V_R = I \cdot R \angle 0°$

$V_C = -jX_C I = I X_C \angle -90°$

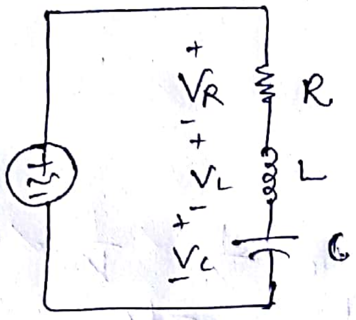


$$\boxed{v = \sqrt{V_R^2 + V_C^2}}$$

$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$

$\cos\phi = \frac{V_R}{V}$ (Leading)

(c) Series R-L-C

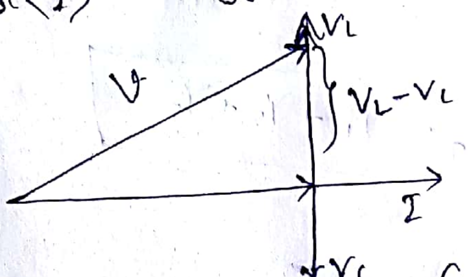


$\bar{V}_R = IR \angle 0°$

$V_L = I X_L \angle +90°$

$V_C = I X_C \angle -90°$.

Case(I) If $X_L > X_C \longrightarrow V_L > V_C$

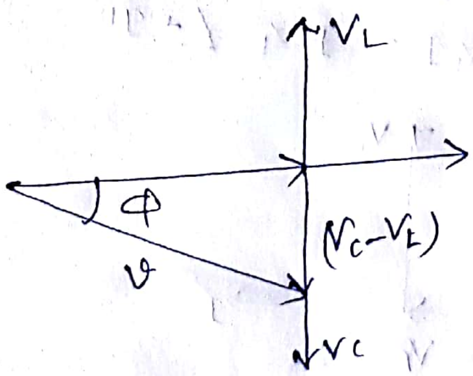


$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

$\cos\phi = \frac{V_R}{V}$ (Lag)

Case(II):- If $X_L < X_C \longrightarrow V_L < V_C$

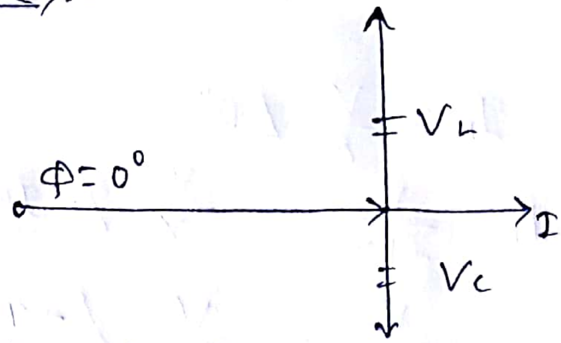


$v = \sqrt{V_R^2 + (V_C - V_L)^2}$

$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$

$\cos\phi = \frac{V_R}{v}$ (Leading)

Case(3):-



$X_C = X_L$

$V_L = V_C$

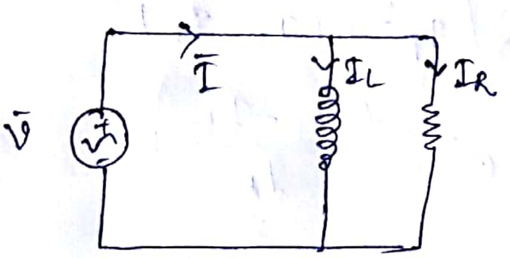
$|V| = V_R$

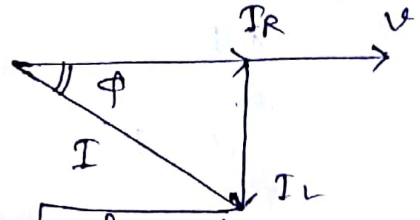
$\phi = 0°$ p.f = 1 (UPF)

⟹ Parallel circuit

$V \to$ ref

(a) Parallel R-L ckt





$|I| = \sqrt{I_R^2 + I_L^2}$

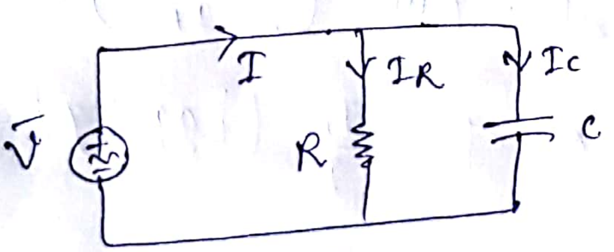
$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$

$I_R = \frac{V}{R} \angle 0°$

$I_L = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90°$ $\cos\phi = \frac{I_R}{I}$ (Lagging)

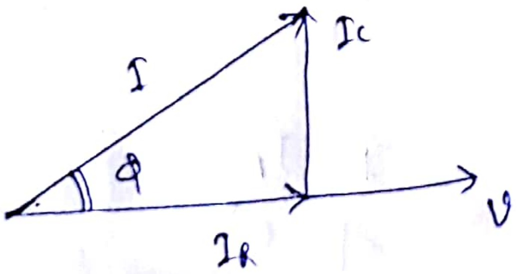
$I_L = \frac{V}{+jX_L} = \frac{V}{X_L} \angle -90°$

(b) Parallel R-C ckt:-



$I_R = \frac{V}{R} \angle 0°$

$I_C = \frac{V}{-jX_C} = \frac{V}{X_C} \angle +90°$

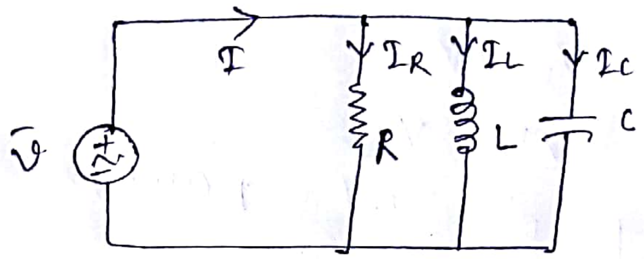


$|I| = \sqrt{I_R^2 + I_C^2}$

$\phi = \tan^{-1}\left[\frac{I_C}{I_R}\right]$

$\cos\phi = \frac{I_R}{I}$ (Leading)

(C) Parallel R-L-C circuit :-

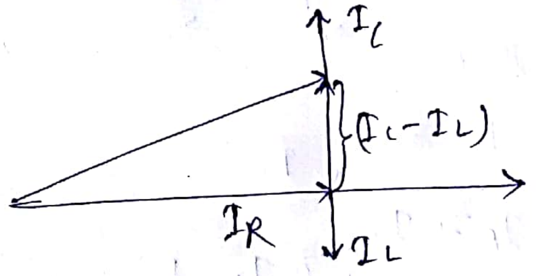


$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{+jX_L} = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{-jX_C} = \frac{V}{X_C} \angle +90^\circ$$

Case(I) :- At $X_L > X_C \longrightarrow I_L < I_C$

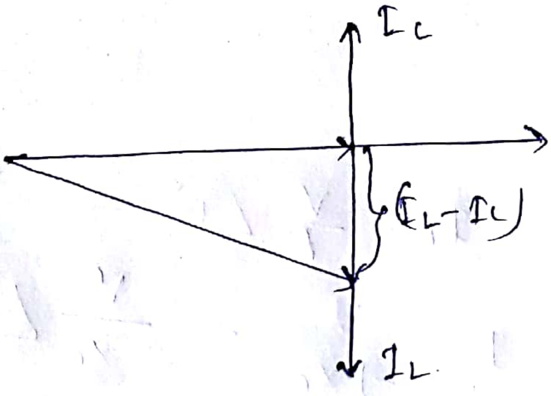


$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

$\cos\phi = I_R / I$ Leading.

Case(2) :- At $X_L < X_C \longrightarrow I_L > I_C$

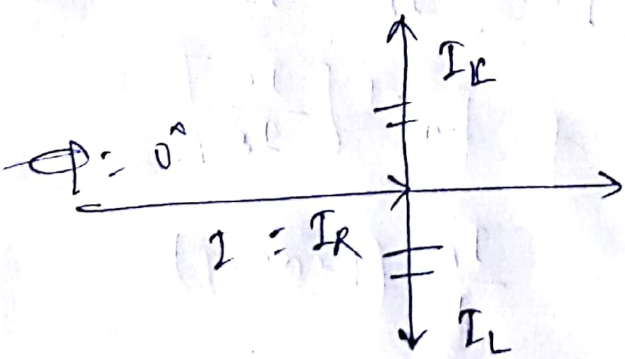


$$|I| = \sqrt{I_R^2 + (I_L - I_C)}$$

$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$\cos\phi = \frac{I_R}{I}$ (Lagging)

Case(III) At $X_L = X_C \longrightarrow I_L = I_C$



$I = I_R$

$\phi = 0^\circ$

p.f = 1 (UPF)