

Measurement of 3- ϕ power

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3- ϕ circuit:

~~Polyphase system~~

Poly-phase system: - More than one phase.

2, 3, 4, 5, 6, 7, 8, 9, ... phases.

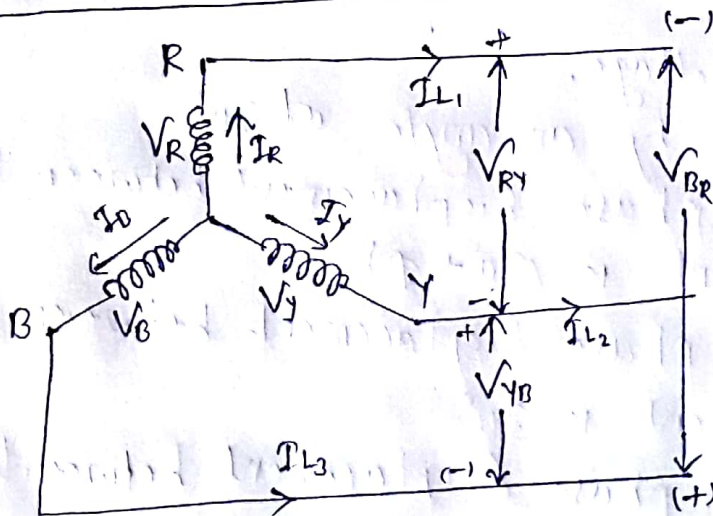
3, 6, 9, 12, ... phases i.e. multiple of 3 are used in industrial applications.

* Main advantages of 3-phase system are.

- Smaller in size.
- Constant output power
- Great output power.
- Superior motor performance.
- Ease in parallel operation.
- Reliability.

Interconnection of system is possible in (Δ & Y)
Total phase shift b/w each phase is $\frac{360^\circ}{3} = 120^\circ$

* STAR Connected Balanced Three phase Supply.



From the figure,

$$I_{L1} = I_{L2} = I_{L3} = I_R = I_Y = I_B$$

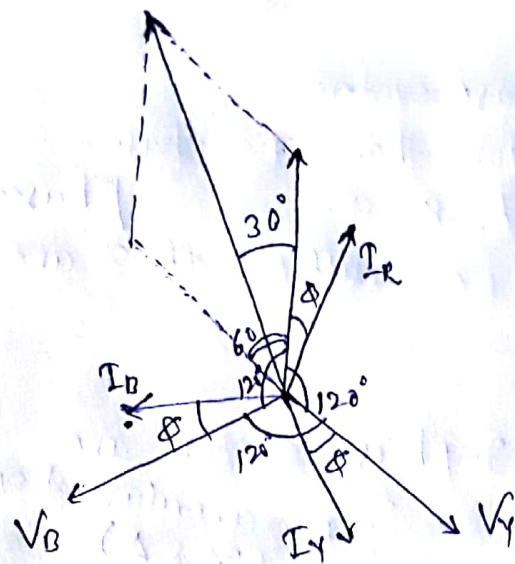
i.e. $I_{ph} = I_L$ in Y -connection.

$$V_{RN} = V_{BN} = V_{YN} = V_{ph} \text{ (Phase voltage)}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L \text{ (Line voltage)}$$

$$\therefore V_{RY} = V_R - V_Y ; V_{YB} = V_Y - V_B ; V_{BR} = V_B - V_R$$

Phasor diagram considering phase voltages as reference.



[for lagging load]

R (+ve)
Y Sequence.
B

$$V_R = V_Y = V_B = V_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_L = \sqrt{3} V_{ph}$$

With respect to phase voltage,

1. Line voltage is leading by an angle of 30°
2. For inductive load, phase displacement between line voltage and line current is $(30^\circ + \phi)$
3. For resistive load, phase displacement between line voltage and line current is 30° .
4. For capacitive load, phase displacement between line voltage and line current is $(30^\circ - \phi)$.

$$V_R = V_{ph} \angle -30^\circ$$

$$V_Y = V_{ph} \angle -120^\circ - 30^\circ$$

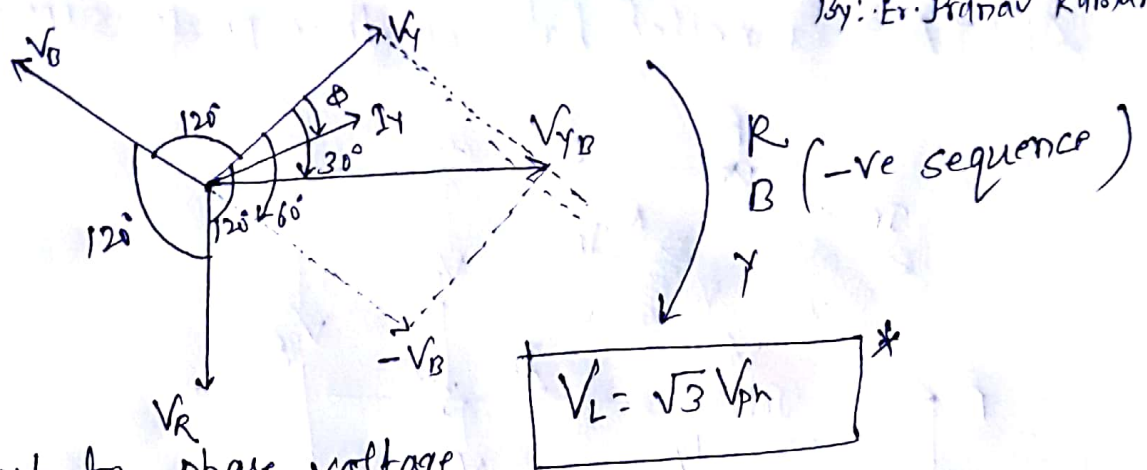
$$V_B = V_{ph} \angle +120^\circ - 30^\circ$$

$$V_{RY} = V_L \angle 0^\circ$$

$$V_{YB} = V_L \angle -120^\circ$$

$$V_{BR} = V_L \angle -240^\circ$$

$$\text{or } V_L \angle +120^\circ$$



* With respect to phase voltage

- ① Line voltage is lagging by an angle of 30° .
- ② For inductive load, phase displacement between line voltage and line current is $(30^\circ - \phi)$.
- ③ For resistive load, the phase displacement between line voltage and line current is 30° .
- ④ For capacitive load, the phase displacement between line voltage and line current is $(30^\circ + \phi)$.

$$V_{RY} = V_L \angle 0^\circ$$

$$V_{YB} = V_L \angle +120^\circ$$

$$V_{BR} = V_L \angle -120^\circ$$

(or) $V_L \angle +240^\circ$

$$V_R = V_{ph} \angle +30^\circ$$

$$V_Y = V_{ph} \angle 120^\circ + 30^\circ$$

$$V_B = V_{ph} \angle -120^\circ + 30^\circ$$

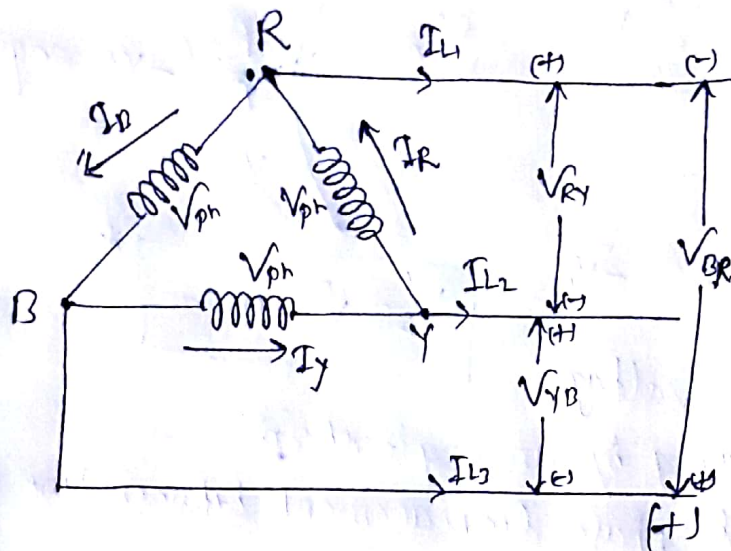
* Power in three phase star connected system.

$$P = 3 V_{ph} I_{ph} = 3 \frac{V_L}{\sqrt{3}} I_L$$

$$P = \sqrt{3} V_L I_L \cos \phi \quad \text{Watts}$$

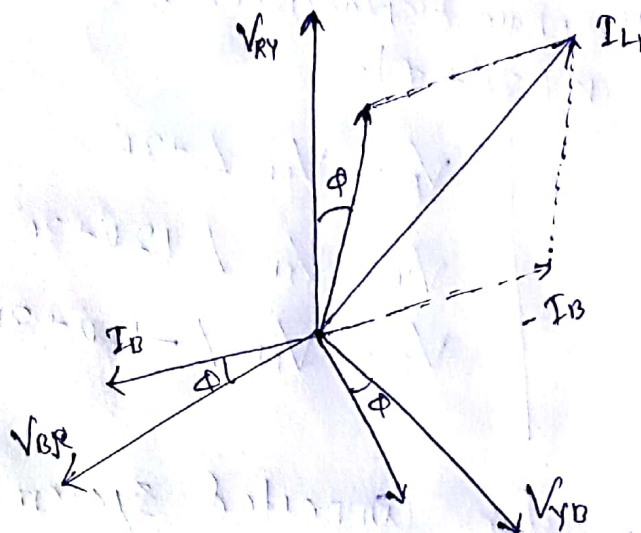
$$Q = \sqrt{3} V_L I_L \sin \phi \quad \text{VARs}$$

DELTA Connected balanced three phase supply.



$$V_{RY} = V_{YB} = V_{BR} = V_L \quad \left\{ \begin{array}{l} I_{L1} = I_R - I_B; I_{L2} = I_Y - I_R; \\ I_{L3} = I_B - I_Y; \end{array} \right.$$

$$\Rightarrow \boxed{V_L = V_{ph}}$$



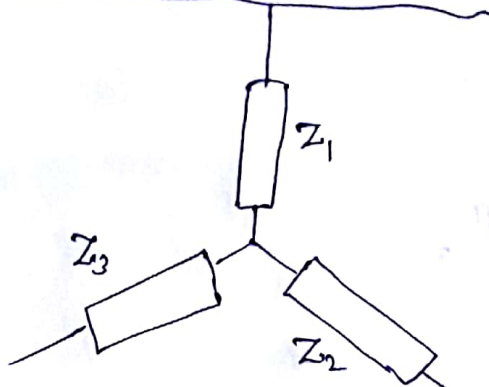
R (+)ve
Y Sequence.
B

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

- (1.) An +ve phase sequence of Δ -connected system with respect to phase current, line current lag by an angle 30° .
- (2.) An (+)ve phase sequence of Δ -connection system, with respect to phase currents, line current Lead by 30° .
- (3.) An (+)ve phase sequence, the phase displacement between line voltage and line current is $L \rightarrow (30^\circ + \phi)$
 $R \rightarrow 30^\circ$, $G \rightarrow (30^\circ + \phi)$
- ④ An (+)ve phase sequence, the phase displacement between line voltage and line current- is $L \rightarrow 30^\circ - \phi$, $R \rightarrow 30^\circ$
 $G \rightarrow 30^\circ + \phi$

* Types of Load and interconnection of three phase circuits.

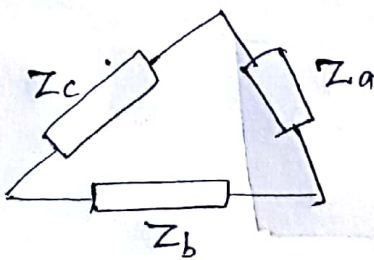
Star connected Load:



$$Z_1 = Z_2 = Z_3 \Rightarrow \text{Balanced}$$

$$Z_1 \neq Z_2 \neq Z_3 \Rightarrow \text{Unbalanced.}$$

Delta connected Load:



$$Z_a = Z_b = Z_c \Rightarrow \text{Balanced}$$

$$Z_a \neq Z_b \neq Z_c \Rightarrow \text{Unbalanced.}$$

* Interconnection between source - load.

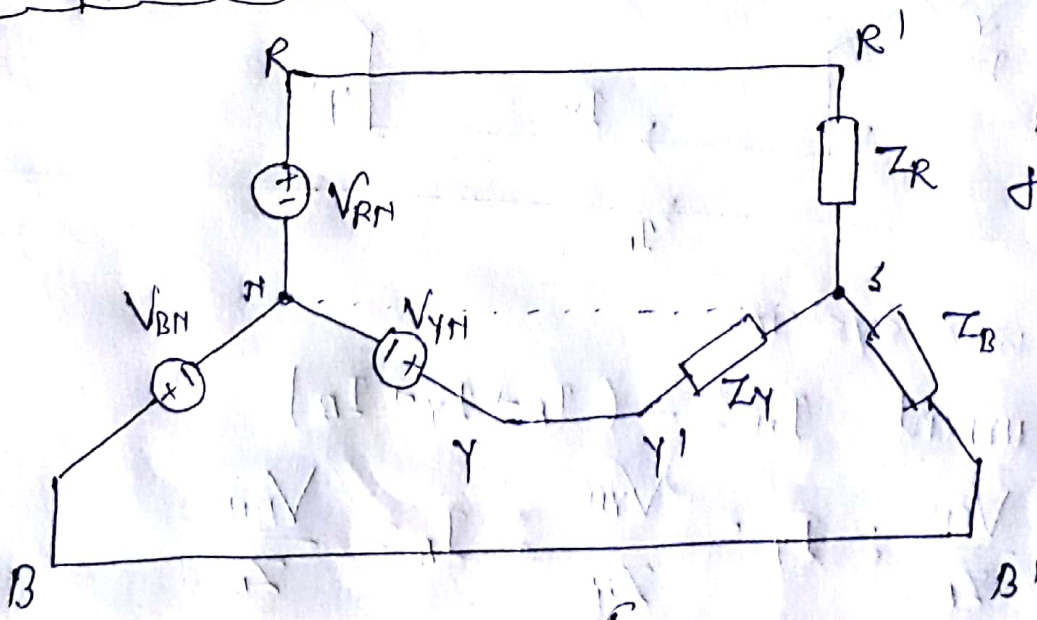
Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

* System source is always balanced
 * Potential at Neutral point of source is always zero whether the load is balanced (or) unbalanced.

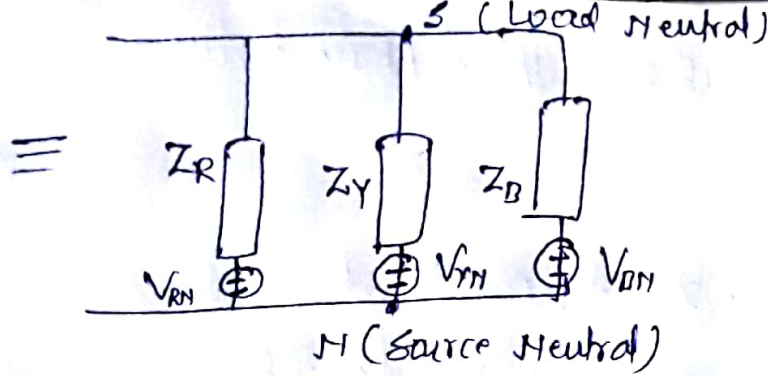
* Load neutral is considered as floating point neutral with

respect to unbalanced load.

3- ϕ , 3-wire



By applying Millman's theorem the above circuit can be simplified to



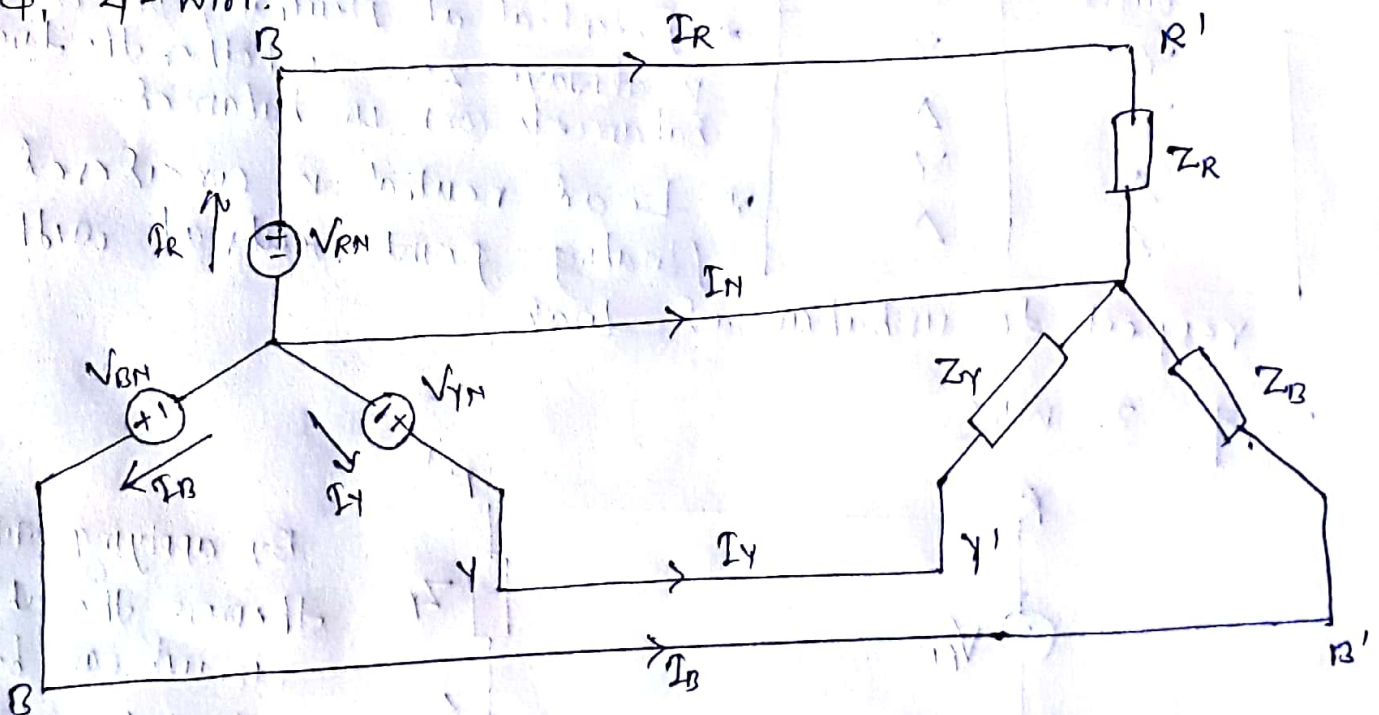
Potential difference between source neutral and load neutral is V_{SN}

$$V_{SN} = \frac{\frac{V_{RN}}{Z_R} + \frac{V_{YN}}{Z_Y} + \frac{V_{BN}}{Z_B}}{\frac{1}{Z_R} + \frac{1}{Z_Y} + \frac{1}{Z_B}}$$

$$V_{SN} = \frac{V_{RN} \cdot Y_R + V_{YN} \cdot Y_Y + V_{BN} \cdot Y_B}{Y_R + Y_Y + Y_B}$$

$$V_{NS} = -V_{SN}$$

3- ϕ , 4-wire



In 3- ϕ , 4-wire system.

$$\text{Neutral current } I_N = -(I_R + I_Y + I_B)$$

$$I_R = \frac{V_{RN}}{Z_R} ; I_Y = \frac{V_{YN}}{Z_Y} ; I_B = \frac{V_{BN}}{Z_B}$$

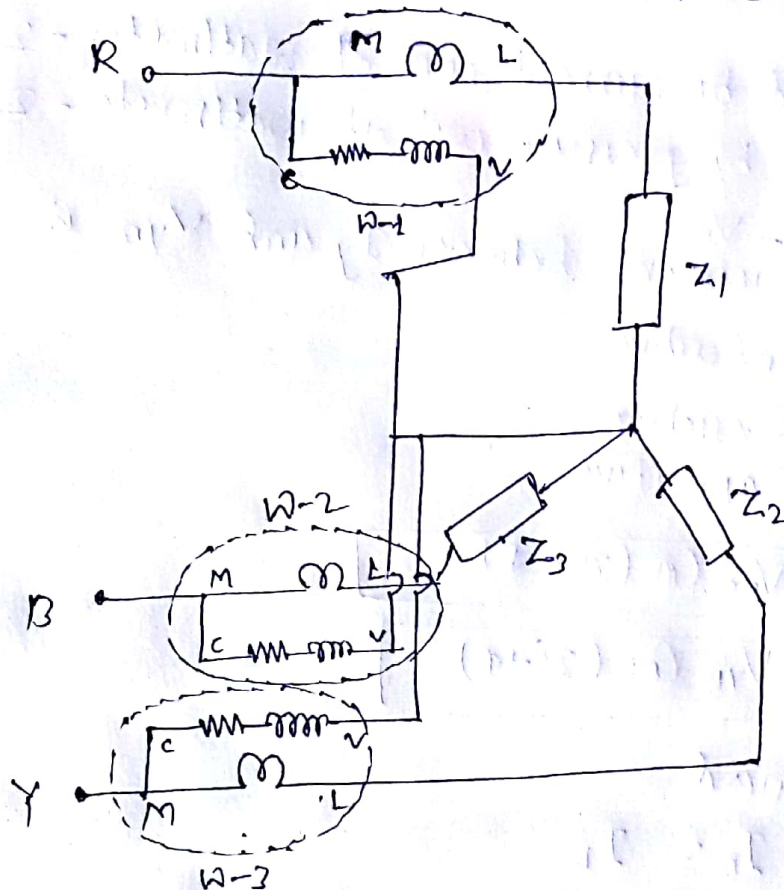
* Measurement of 3- ϕ power

- 3-wattmeter method used for balanced or Unbalanced loads.
- 2-wattmeter method used for balanced or Unbalanced loads.
- 1-wattmeter method used for balanced load only.

3- ϕ , 3-wattmeter method:

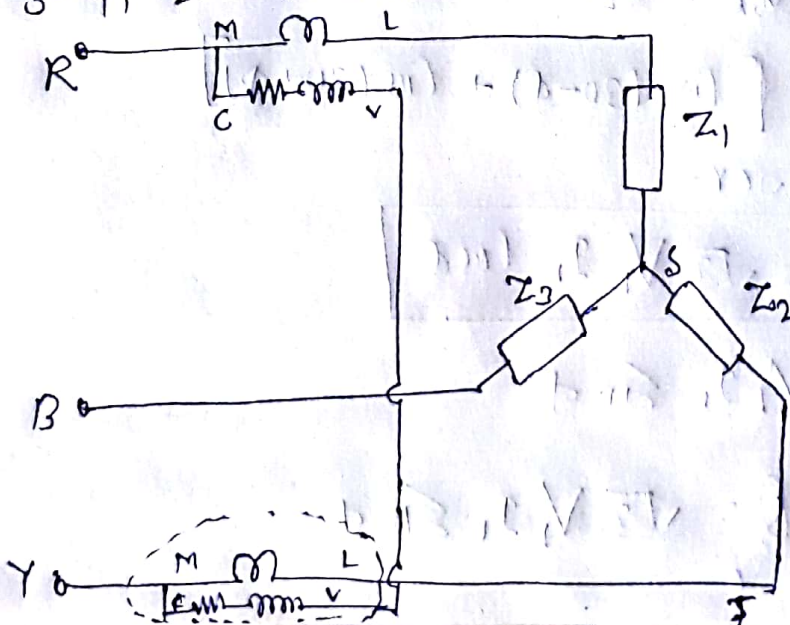
In a commercial wattmeter we can identify the terminal connections are.

M $\rightarrow$ Mains, L $\rightarrow$ Load, C $\rightarrow$ Common, V $\rightarrow$ Voltage.



Total three phase power = $W_1 + W_2 + W_3$

• 3- ϕ , 2-wattmeter method:



Wattmeter W_1 :

- (i) Current measured by current coil of wattmeter - 1 = I_R
- (ii) Voltage measured by pressure coil of wattmeter - 1
 $V_{RB} = V_R - V_B$
- (iii) The phase displacement between ' I_R ' and ' V_{RB} ' is equal to.
 - $(30^\circ - \phi) \rightarrow$ Inductive
 - $30^\circ \rightarrow$ Resistive
 - $(30^\circ + \phi) \rightarrow$ Capacitive.

Wattmeter W_2 :

- (i) Current measured by current coil of wattmeter - 2 is = I_Y
- (ii) Voltage measured by pressure coil of wattmeter - 2
 $V_{YB} = V_Y - V_B$
- (iii) The phase displacement between I_Y and V_{YB} is
 - $(30^\circ + \phi) \rightarrow$ Inductive
 - $30^\circ \rightarrow$ Resistive
 - $(30^\circ - \phi) \rightarrow$ Capacitive.

So

$$W_1 = I_R V_{RB} \cos(30^\circ - \phi)$$
$$W_2 = I_Y V_{YB} \cos(30^\circ + \phi)$$

for balanced load

$$I_R = I_Y = I_B = I_L$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$W_1 + W_2 = V_L I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)]$$

So, total active power.

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi$$

$$\text{Also } W_1 - W_2 = V_L I_L \sin \phi$$

$$\text{But reactive power } Q = \sqrt{3} V_L I_L \sin \phi$$

$$\therefore Q = \sqrt{3} (W_1 - W_2)$$

$$\Rightarrow \frac{Q}{P} = \frac{\sqrt{3} V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi} = \frac{\sqrt{3} (W_1 - W_2)}{W_1 + W_2}$$

$$\Rightarrow \tan \phi = \frac{\sqrt{3} (W_1 - W_2)}{W_1 + W_2}$$

$$\therefore \phi = \tan^{-1} \left[\frac{\sqrt{3} (W_1 - W_2)}{W_1 + W_2} \right]$$

Effect of power factor on wattmeter Reading.

$$W_1 = V_L I_L \cos (30^\circ - \phi)$$

$$W_2 = V_L I_L \cos (30^\circ + \phi)$$

Case (i) If $\phi = 0^\circ$,

$$W_1 = V_L I_L \cos 30^\circ \text{ and } W_2 = V_L I_L \cos 30^\circ$$

$\therefore W_1 = W_2 \rightarrow$ If power factor angle is 0° both wattmeter reading is equal.

Case (ii) If $\phi = 30^\circ$

$$W_1 = V_L I_L \cos (30^\circ - 30^\circ) = V_L I_L$$

$$W_2 = V_L I_L \cos (30^\circ + 30^\circ) = \frac{V_L I_L}{2}$$

$$\therefore W_2 = \frac{W_1}{2}$$

If power factor angle is 30° one wattmeter readings is half of the other

Case (iii) If $\phi = 60^\circ$

$$W_1 = V_L I_L \cos (30^\circ - 60^\circ) = \frac{\sqrt{3}}{2} V_L I_L$$

$$W_2 = V_L I_L \cos (30^\circ + 60^\circ) = 0$$

If power factor angle is 60° , one of the wattmeter reading is zero.

Case (iv) If $\phi = 90^\circ$

$$W_1 = V_L I_L \cos (30^\circ - 90^\circ) = \frac{V_L I_L}{2}$$

$$W_2 = V_L I_L \cos (30^\circ + 120^\circ) = -\frac{V_L I_L}{2}$$

$$W_2 = -W_1$$

If power factor angle is 90° one wattmeter reading is negative.