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Convolution Property of Z-Transform

We know that the convolution of two discrete time signals $x_1(n)$ and $x_2(n)$ is given by

$$x(n) = x_1(n) * x_2(n)$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k) = \sum_{k=-\infty}^{\infty} x_1(n-k) x_2(k)$$

Now, the convolution of z-Transform state that

If $z[x_1(n)] = X_1(z)$

$z[x_2(n)] = X_2(z)$.

Than $x(n) = z[x_1(n) * x_2(n)] = X(z)$

$\therefore X(z) = X_1(z) \cdot X_2(z)$

Proof : we know that

$$x_1(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k) = \sum_{k=-\infty}^{\infty} x_1(n-k) x_2(k) \quad \text{--- (1)}$$

Also, we know that

$$z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{--- (2)}$$

from eqn (1) & (2) we have

$$z[x_1(n) * x_2(n)] = \sum_{n=-\infty}^{\infty} [x_1(n) * x_2(n)] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{ \sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k) \right\} z^{-n} = \sum_{n=-\infty}^{\infty} \left\{ x_1(n) \sum_{n=-\infty} x_2(n-k) z^{-n} \right.$$

let $n-k = m$, we get

$$z[x_1(n) * x_2(n)] = \sum_{k=-\infty}^{\infty} \left\{ x_1(n) \sum_{n=-\infty}^{\infty} x_2(m) z^{-m-k} \right.$$

$$= \sum_{k=-\infty}^{\infty} x_1(k) z^{-k} \cdot \sum_{m=-\infty}^{\infty} x_2(m) z^{-m}$$

$$= X_1(z) \cdot X_2(z).$$

$z[x_1(n) * x_2(n)] = X_1(z) \cdot X_2(z)$ Proved

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(1) Find the final value theorem $x(\infty)$ of $x(z)$ is given by

(a) $\dfrac{z+1}{(z-0.6)^2}$ (b) $\dfrac{z+2}{4(z-1)(z+0.7)}$

Sol. (a) $x(z) = \dfrac{z+1}{(z-0.6)^2}$

$$x(\infty) = \lim_{z \to 1} (z-1)\, x(z)$$

$$= \lim_{z \to 1} (z-1) \times \frac{z+1}{(z-0.6)^2}$$

$$= 0. \text{ Ans}$$

(b) $x(z) = \dfrac{z+2}{4(z-1)(z+0.7)}$

$$(z-1)\, x(z) = \frac{z+2}{4(z+0.7)}$$

$$x(\infty) = \lim_{z \to 1} (z-1)\, x(z)$$

$$= \lim_{z \to 1} \left[\frac{z+2}{4(z+0.7)}\right]$$

$$= \frac{3}{6.8} = 0.44. \text{ Ans}$$

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(2) Find the $x(0)$ of $x(z)$ is given by

(a) $\dfrac{z^2+2z+2}{(z+1)(z+0.2)}$ (b) $\dfrac{z+3}{(z+1)(z+2)}$

(a) $x(z) = \dfrac{z^2+2z+2}{(z+1)(z+0.2)} = \dfrac{1+\frac{2}{z}+\frac{2}{z^2}}{1+\left(\frac{1}{z}\right)\left[1+\left(\frac{0.5}{z}\right)\right]}$

$$x(0) = \lim_{z \to \infty} x(z) = \lim_{z \to \infty} \frac{\left[1+\frac{2}{z}+\frac{2}{z^2}\right]}{\left[1+\left(\frac{1}{z}\right)\left[1+\left(\frac{0.5}{z}\right)\right]\right]} = 1. \text{ Ans}$$

(b) $x(z) = \dfrac{z+3}{(z+1)(z+2)}$

$$= \frac{z\left[1+\left(\frac{3}{z}\right)\right]}{z^2\left[1+\left(\frac{1}{z}\right)\left[1+\left(\frac{2}{z}\right)\right]\right]} = \frac{1}{z} \frac{1+\left(\frac{3}{z}\right)}{\left[1+\left(\frac{1}{z}\right)\left[1+\left(\frac{2}{z}\right)\right]\right]}$$

$$x(0) = \lim_{z \to \infty} x(z) = \lim_{z \to \infty} \frac{1}{z} \cdot \frac{1+\left(\frac{3}{z}\right)}{\left[1+\left(\frac{1}{z}\right)\left[1+\left(\frac{2}{z}\right)\right]\right]} = 0 \text{ Ans}$$

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Q. Find the convolution of the sequence $x_1(n) = \left(\frac{1}{3}\right)^n u(n)$ and $x_2(n) = \left(\frac{1}{5}\right)^n u(n)$ using (a) convolution property of Z-Transform and (b) Time domain method.

Soln

(a) Convolution property of Z-Transform.

given $x_1(n) = \left(\frac{1}{3}\right)^n u(n)$

and $x_2(n) = \left(\frac{1}{5}\right)^n u(n)$.

$$\therefore X_1(z) = Z\left[\left(\frac{1}{3}\right)^n u(n)\right]$$

$$= \frac{1}{1-\left(\frac{1}{3}\right)z^{-1}} = \frac{z}{z-(1/3)} \qquad |z| > 1/3$$

and $X_2(z) = Z\left[\left(\frac{1}{5}\right)^n u(n)\right]$

$$= \frac{1}{1-\left(\frac{1}{5}\right)z^{-1}} = \frac{z}{z-\left(\frac{1}{5}\right)} \qquad |z| > 1/5$$

We know that

$$x(n) = x_1(n) * x_2(n)$$

$$\therefore Z[x(n)] = X(z) = Z[x_1(n) * x_2(n)] = X_1(z)X_2(z)$$

$$Z[x_1(n) * x_2(n)] = \frac{z}{z-(1/3)} \cdot \frac{z}{z-\left(\frac{1}{5}\right)}$$

Taking ILT

$$x(n) = Z^{-1}\left[\frac{z}{z-(1/3)} \cdot \frac{z}{z-(1/5)}\right]$$

$$= Z^{-1}\left[\frac{5}{2} \cdot \frac{z}{z-(1/3)} - \frac{3}{2} \cdot \frac{z}{z-\left(\frac{1}{5}\right)}\right]$$

$$= \frac{5}{2}\left(\frac{1}{3}\right)^n u(n) - \frac{3}{2}\left(\frac{1}{5}\right)^n u(n) \quad \text{Ans}$$

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(b) Time domain method.

Given, $x_1(n) = \left(\frac{1}{3}\right)^n u(n)$

$x_2(n) = \left(\frac{1}{5}\right)^n u(n)$

$x(n) = x_1(n) * x_2(n) = \sum_{k=0}^{n} x_1(k)\, x_2(n-k)$

$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k u(k) \left(\frac{1}{5}\right)^{n-k} u(n-k)$

$= \sum_{k=0}^{n} \left(\frac{1}{3}\right)^k \left(\frac{1}{5}\right)^n \left(\frac{1}{5}\right)^{-k}$

$= \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} \left(\frac{1}{3} \times \frac{5}{1}\right)^k$

$= \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} \left(\frac{5}{3}\right)^k$

$= \left(\frac{1}{5}\right)^n \left[\frac{1-\left(\frac{5}{3}\right)^{n+1}}{1-\left(\frac{5}{3}\right)}\right]$

$= -\frac{3}{2}\left(\frac{1}{5}\right)^n \left\{\left(\frac{1}{5}\right)^n \left[1-\left(\frac{5}{3}\right)^n \frac{5}{3}\right]\right\}$

$= -\frac{3}{2}\left(\frac{1}{5}\right)^n + \frac{3}{2}\left(\frac{1}{5}\right)^n \left(\frac{5}{3}\right)^n \cdot \frac{5}{3}$

$= \frac{5}{2}\left(\frac{1}{3}\right)^n u(n) - \frac{3}{2}\left(\frac{1}{5}\right)^n u(n)$ Ans

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Assignment (signal and systems)

(1) Determine the value of signal $x(n)$ at $n=0$ and $n=\infty$ of

$$X(z) = \frac{2z^2+0.25}{(z+0.25)(z-1)}$$

(2) Determine the z-transforms of

$$x(n) = \left[3\left(\frac{4}{5}\right)^n - \left(\frac{2}{3}\right)^{2n}\right]u(n)$$

(3) Find the z-transform of

$$X(z) = \frac{z^3}{(z+1)(z-1)^2}$$

(4) Using z-transform method with min^m number of delay unit. Assume initial cond^n to zero.

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n) + 2x(n-1)$$

(5) Find the inverse z-transform of

$$X(z) = \frac{2z^3-5z^2+z+3}{(z-1)(z-2)} \qquad |z|<1$$

* Submit the assignment on 10.04.2020 (or before).

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