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(1) Find the 1st five terms of the sampled function $f^*(t)$ for which $F(z)$ is given by

$$F(z) = \frac{z^2+8z+12}{z^2+2z+3}$$ Using long division method.

Soln: given $$F(z) = \frac{z^2+8z+12}{z^2+2z+3} \quad \text{— (1)}$$

The order of numerator and denominator are same

i.e $\boxed{n = m}$

Divide numerator and denominator eqn by z^2

$$F(z) = \frac{z^2+8z+12/z^2}{z^2+2z+3/z^2}$$

$$F(z) = \frac{1+8z^{-1}+12z^{-2}}{1+2z^{-1}+3z^{-2}} \quad \text{— (2)}$$

$$1+2z^{-1}+3z^{-2}\,\big)\; 1+8z^{-1}+12z^{-2}\;\big(\,1+6z^{-1}-3z^{-2}-12z^{-3}+33z^{-4}-\cdots$$

$$-\;(1+2z^{-1}+3z^{-2})$$

$$6z^{-1}+9z^{-2}$$

$$-\;(6z^{-1}+12z^{-2}+18z^{-3})$$

$$-3z^{-2}-18z^{-3}$$

$$-\;(-3z^{-2}-6z^{-3}-9z^{-4})$$

$$-12z^{-3}+9z^{-4}$$

$$-\;(-12z^{-3}-24z^{-4}-36z^{-5})$$

$$33z^{-4}+36z^{-5}$$

$$-\;(33z^{-4}+66z^{-5}+99z^{-6})$$

$$-30z^{-5}-99z^{-6}$$

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eqn (2) can be written as

$$F(z) = 1+6z^{-1}-3z^{-2}-12z^{-3}+33z^{-4}- - - \quad$$

we know that

$$F(z) = \sum_{k=0}^{\infty} f(kT)z^{-k}$$

$$= f(0)+f(T)z^{-1}+f(2T)z^{-2}+f(3T)z^{-3}+ - - \quad \text{— (4)}$$

Comparision eqn (3) & (4) we get

$f(0)=1,\ f(T)=6,\ f(2T)=-3,\ f(3T)=-12,\ f(4T)=33$

Taking inverse Z-Transform of eqn (3).

$$f^*(t) = \delta(t)+6\delta(t-T)-3\delta(t-2T)-12\delta(t-3T)$$
$$+33\delta(t-4T).$$

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Solⁿ of Difference eqⁿ:

How to solve the Difference eqⁿ. from the Z-Transform.

Prob(1) Solve the difference eqⁿ. by using Z-Transform method.

$$x(k+2)+3x(k+1)+2x(k)=0$$
with $x(0)=0$ and $x(1)=1$

Solⁿ. given difference eqⁿ. in discrete time function.

$$x(k+2)+3x(k+1)+2x(k)=0 \quad -(1)$$

Applying Z-Transform.

$$Z[x(k+2)]+Z[3x(k+1)]+Z[2x(k)]=0.$$

$$[z^2X(z)-z^2x(0)-zx(1)]+3[zX(z)-zx(0)]+2X(z)=0$$

put $x(0)=0,\ x(1)=1.$

$$z^2X(z)+3zX(z)+2X(z)-z^2x(0)-zx(1)-3zx(0)=0$$

$$[z^2+3z+2]X(z)=0-z\times1-0=0$$

$$(z^2+3z+2)X(z)=z$$

$$X(z)=\frac{z}{z^2+3z+2}$$

$$=\frac{z}{z^2+2z+z+2}=\frac{z}{z(z+2)+1(z+2)}$$

$$X(z)=\frac{z}{(z+1)(z+2)} \quad -(2)$$

Taking inverse Z-Transform of eqⁿ (2) using Residue Method.

$$x(kT)=\text{Residue of } F(z).z^{k-1} \text{ at } z=-1 + \text{Residue of } F(z).z^{k-1} \text{ at } z=-2$$

$$=\lim_{z=-1}\left[\cancel{(z+1)}\cdot\frac{z}{\cancel{(z+1)}(z+2)}\cdot z^{k-1}\right]+\lim_{z=-2}\left[\cancel{(z+2)}\frac{z}{(z+1)\cancel{(z+2)}}z^{k-1}\right]$$

$$=\lim_{z=-1}\left[\frac{z}{z+2}\cdot z^{k-1}\right]+\lim_{z=-2}\left[\frac{z.z^{k-1}}{z+1}\right]$$

$$=\lim_{z=-1}\left[\frac{z^k}{z+2}\right]+\lim_{z=-2}\left[\frac{z^k}{z+1}\right]$$

$$=\frac{(-1)^k}{(-1+2)}+\frac{(-2)^k}{(-2+1)} \quad -(3)$$

$$\boxed{\therefore x(kT)=(-1)^k-(-2)^k}$$

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When $k=0$ $x(0) = (-1)^0 - (-2)^0 = 0$ i.e $x(0) = 0$
When $k=1$ $x(T) = (-1)^1 - (-2)^1 = -1+2$ $x(T) = 1$
When $k=2$ $x(2T) = (-1)^2 - (-2)^2 = 1-4$ $x(2T) = -3$
When $k=3$ $x(3T) = (-1)^3 - (-2)^3 = -1+8$ $x(3T) = 7$
When $k=4$ $x(4T) = (-1)^4 - (-2)^4 = 1-16$ $x(4T) = -15$

Ans

(2) Find the response of the system,

$$x(k+2) - 5x(k+1) + 6x(k) = u(k)$$

with $x(0) = 0$, $x(1) = 1$
and $u(k) = 1$, for $k = 0, 1, 2, 3$ -- by z-Transform method

Soln $x(k+2) - 5x(k+1) + 6x(k) = u(k)$
Taking z-Transform of both side of the given eqn we get

$$z[x(k+2) - 5x(k+1) + 6x(k)] = z[u(k)]$$

$$z[x(k+2)] - z[5x(k+1)] + z[6x(k)] = z[u(k)]$$

$$[z^2x(z) - z^2x(0) - zx(1)] - 5[zx(z) - zx(0)] + 6x(z) = z\,u(k)$$

Put $x(0)=0$, $x(1)=1$

$$[z^2x(z) - 0 - z] - [5zx(z) - 0] + 6x(z) = \frac{z}{z-1}$$

$$[z^2 - 5z + 6]x(z) = \frac{z}{z-1} + z$$

$$(z^2 - 5z + 6)x(z) = \frac{z + z(z-1)}{z-1}$$

$$(z^2 - 5z + 6)x(z) = \frac{\cancel{z} + z^2 - \cancel{z}}{z-1}$$

$$x(z) = \frac{z^2}{(z-1)(z^2-5z+6)}$$

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-2)(z-3)}$$

$$= \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$A = 1/2, \quad B = -2, \quad C = 3/2$$

$$x(z) = \frac{1}{2} \cdot \frac{z}{z-1} - 2\frac{z}{z-2} + \frac{3}{2} \cdot \frac{z}{z-3}$$

Taking IZT

$$f(kT) = \frac{1}{2} \cdot (1) - 2 \cdot (2)^k + \frac{3}{2} \cdot (3)^k$$ Ans

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(3) Q. Find the response $x(k)$ of the following

$$x(k+2) - 3x(k+1) + 2x(k) = u(k)$$

with $x(k) = 0$, for $k \leq 0$, $u(0) = 1$

given $x(0) = 0$ $\quad$ $u(k) = 0$, for $k > 1$ or $k < 1$.

$x(1) = 1$

Soln. 2nd order discrete function is given

$$x(k+2) - 3x(k+1) + 2x(k) = u(k) \quad \text{--- (1)}$$

Taking Z-Transform of eqn (1)

$$Z[x(k+2)] - Z[3x(k+1)] + Z[2x(k)] = Z[u(k)]$$

$$\left[z^2 x(z) - z^2 x(0) - z x(1)\right] - 3\left[z x(z) - z x(0)\right]$$

$$+ 2x(z) = Z[u(k)]$$

$$(z^2 - 3z + 2)\, x(z) - z^2 x(0) - z x(1) + 3z x(0) = Z[u(k)]$$

putting the value of $x(0) = 0$

$x(1) = 1$.

$$(z^2 - 3z + 2)\, x(z) - z^2 \times 0 - z \times 1 + 3 \times 0 = Z[u(k)]$$

$$(z^2 - 3z + 2)\, x(z) - z = Z[u(k)]$$

$$Z[u(k)] = \sum_{k=0}^{\infty} u(k) z^{-k}$$

since $u(k) = 0$, $k < 1$, $k \geq 1$

$$Z[u(k)] = u(0) z^{-0}$$

$$= 1$$

$$(z^2 - 3z + 2)\, x(z) - z = 1.$$

$$(z^2 - 3z + 2)\, x(z) = 1 + z$$

$$x(z) = \frac{z+1}{z^2 - 3z + 2}$$

$$x(z) = \frac{z+1}{(z-1)(z-2)}$$

Taking Inverse Z-Transform

using Residue method.

$$x(kT) = \text{Residue of } F(z) \cdot z^{k-1} \text{ at } z = 1$$

$$+ \text{Residue of } F(z) \cdot z^{k-1} \text{ at } z = 2.$$

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$$x(kT) = \lim_{z \to 1}\left[\cancel{(z-1)} \frac{z+1}{\cancel{(z-1)}(z-2)} \cdot z^{k-1}\right] + \lim_{z \to 2}\left[\cancel{(z-2)} \frac{z+1}{(z-1)\cancel{(z-2)}} z^{k-1}\right]$$

$$= \lim_{z \to 1}\left[\frac{(z+1) \cdot z^{k-1}}{z-2}\right] + \lim_{z \to 2}\left[\frac{z+1}{z-1} z^{k-1}\right]$$

$$= \frac{(1+1)1^{k-1}}{(1-2)} + \frac{(2+1)}{(2-1)} \cdot 2^{k-1}$$

$$\boxed{x(kT) = (-2)^{k-1} + 3(2)^{k-1}} \quad \text{— (3)}$$

when $k=0$ $\quad x(0) = (-2)1^{0-1} + 3 \cdot 2^{0-1} = -2 + 3 \cdot \frac{1}{2} = -0.5$

when $k=1$ $\quad x(T) = (-2)1^{1-1} + 3 \cdot 2^{1-1} = -2 + 3 = 1$

when $k=2$ $\quad x(2T) = (-2)1^{2-1} + 3 \cdot 2^{2-1} = -2 + 6 = 4.$

when $k=3$ $\quad x(3T) = (-2)1^{3-1} + 3 \cdot 2^{3-1} = -2 + 12 = 10.$

$$\left.\begin{aligned} x(0) &= -0.5 \\ x(T) &= 1 \\ x(2T) &= 4 \\ x(3T) &= 10 \end{aligned}\right\} \text{Ans}$$

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