Step response of series R-L-C ckt :

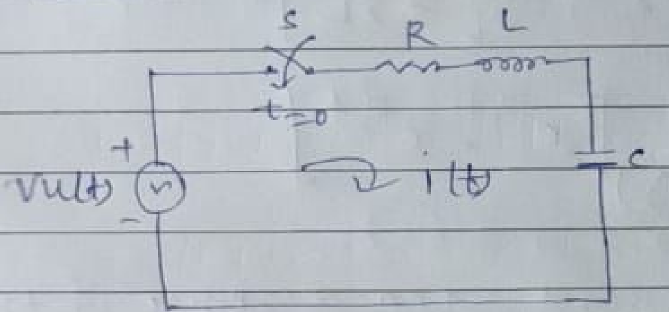


Fig: R-L-C ckt

Fig. R-L-C ckt shown.

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt = v u(t)$$

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt = v u(t)$$

Taking Laplace Transform on both sides, we get

$$RI(s) + L\left[sI(s) - i(0^+)\right] + \frac{1}{C}\frac{q(0^+)}{s} + \frac{1}{C}\frac{I(s)}{s} = \frac{V}{s}$$

$$RI(s) + L\left[sI(s) - i(0^+)\right] + \frac{1}{C}\frac{q(0^+)}{s} + \frac{1}{C}\frac{I(s)}{s} = \frac{V}{s}$$

Where $i(0^+)$ is the initial current through the inductor and $q(0^+)$ is the initial charge on the capacitor. Neglecting the all initial condⁿ, we get

$$RI(s) + LsI(s) + \frac{1}{Cs}I(s) = \frac{V}{s}$$

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \frac{V}{s}$$

$$I(s)\left[Ls + R + \frac{1}{Cs}\right] = \frac{V}{s}$$

$$I(s)\left[\frac{LCs^2 + RCs + 1}{C\cancel{s}}\right] = \frac{V}{\cancel{s}}$$

$$I(s) = \frac{V \times C}{\left[LCs^2 + RCs + 1\right]}$$

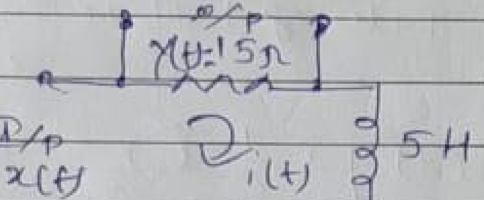
$$= \frac{V}{Ls^2 + Rs + \left(\frac{1}{C}\right)}$$

$$= \frac{V}{L\left[s^2 + \left(\frac{R}{L}\right)s + \left(\frac{1}{LC}\right)\right]} = \frac{V}{L\left[(s-P_1)(s-P_2)\right]}$$

Where $P_1, P_2 = -\frac{R}{2L} \pm \frac{1}{2L}\sqrt{R^2 - 4\frac{L}{C}}$

$$i(t) = \frac{V/L}{(P_1 - P_2)}\left[e^{P_1 t} - e^{P_2 t}\right]$$

Prob ① Find the unit step response of the ckt shown in fig.



$x(t) = 15\, i(t) + 5\frac{di(t)}{dt}$

Taking Laplace Transform on both sides, we have

$$X(s) = 15 I(s) + 5\left[s I(s) - i(0^+)\right]$$

$$X(s) = 15 I(s) + 5 s I(s)$$

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$$X(s) = I(s)\left[15 + 5s\right]$$

$$X(s) = 5 I(s)\left[3 + s\right]$$

$$\therefore I(s) = \frac{X(s)}{5(s+3)}$$

I/p $\rightarrow x(t)$

O/p $\rightarrow r(t)$

O/p value is 15 Ω

$\frac{\text{output}}{\text{Input}}$ = Transfer = $\frac{15}{x(t)}$

output $R(s) = I(s) \times 15$

$$= \cancel{15}^{\,3} \times \frac{X(s)}{\cancel{5}(s+3)}$$

$$= \frac{3}{s+3} X(s)$$

For unit step response

$x(t) = u(t)$

$X(s) = \frac{1}{s}$.

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$$Y(s) = \frac{3}{s+3} \cdot X(s)$$

$$= \frac{3}{s(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+3}$$ By Part of fraction find the value of A & B.

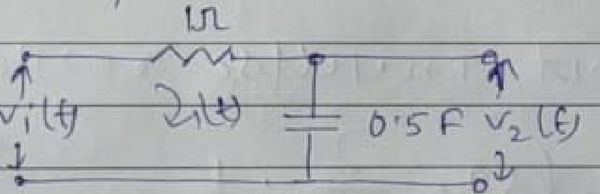
$$= \frac{1}{s} - \frac{1}{s+3}$$

Taking ILT, the unit step response of the given ckt is

$$y(t) = u(t) - e^{-3t}u(t)$$

$$= [1 - e^{-3t}]u(t) \quad \underline{\underline{An}}$$

(2) A continous, causal and linear time invariant system is shown in fig. Determine the unit impulse response and unit step response of the system.



$$v_1(t) = i(t) + \frac{1}{C}\int i\,dt$$

$$v_1(t) = i(t) + \frac{1}{0.5}\int i\,dt$$

Taking laplace Transform on both sides, we have

$$V_1(s) = I(s) + 2\left[\frac{I(s)}{s} + \frac{q(0^+)}{s}\right] \Rightarrow 0$$

$$V_1(s) = I(s) + 2\frac{I(s)}{s}$$

$$V_1(s) = I(s)\left[1 + \frac{2}{s}\right] = I(s)\left[\frac{s+2}{s}\right]$$

$$I(s) = V_1(s) \times \frac{s}{s+2}$$

$v_2(t) = \frac{1}{C}\int i(t)\,dt$

Taking laplace

$= \frac{1}{0.5}\int i(t)\,dt$

$V_2(s) = \frac{2}{s} I(s)$

Put the value of $I(s)$.

$\therefore V_2(s) = \frac{2}{\cancel{s}} \times V_i(s) \times \frac{\cancel{s}}{s+2}$

$V_2(s) = \frac{2}{s+2} V_i(s)$

If the Input is a unit impulse

i.e $v_1(t) = \delta(t)$

$V_1(s) = 1.$

The unit Impulse response

$V_2(s) = \frac{2}{s+2} \cdot 1$

$V_2(s) = \frac{2}{s+2}$

Taking ILT $v_2(t) = 2e^{-2t}u(t)$

If the input is a unit step

$v_1(t) = u(t)$

Then $V_1(s) = \frac{1}{s}$.

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The unit step response is

$$V_2(s) = V_1(s) \times \frac{2}{s+2}$$

$$= \frac{1}{s} \times \frac{2}{s+2}$$

$$= \frac{2}{s(s+2)}$$

$$= \frac{A}{s} + \frac{B}{s+2}$$

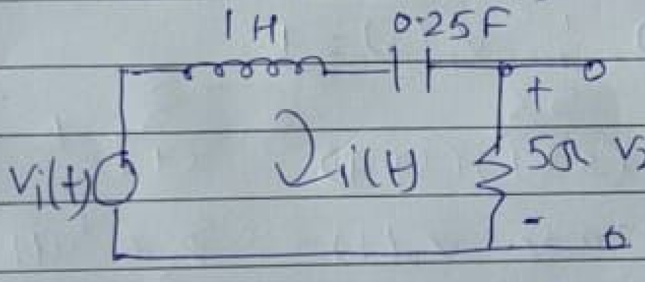
After find the value of A & B from the partial fraction expansion

$$V_2(s) = \frac{1}{s} - \frac{1}{s+2}$$

$$v_2(t) = u(t) - e^{-2t}u(t)$$

(5) obtain the voltage across the resistor in the ckt shown in fig. as a function of time for $t>0$ Assume that $i(0)=0$, $v_c(0)=0$

(a) $v_1(t) = \delta(t)$ (b) $v_1(t) = u(t)$



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$$v_1(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt + R I(t)$$

$$v_1(t) = \frac{di(t)}{dt} + \frac{1}{0.25}\int i\,dt + 5I(t)$$

Taking laplace on both sides

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$$V_1(s) = sI(s) - i(0^+) + 4\left[\frac{I(s)}{s} + \frac{q(0^+)}{s}\right] + 5I(s)$$

Neglecting all initial condⁿ we have

$$V_1(s) = sI(s) + 4\frac{I(s)}{s} + 5I(s)$$

$$= I(s)\left[s + \frac{4}{s} + 5\right]$$

$$V_1(s) = I(s)\left[\frac{s^2+5s+4}{s}\right]$$

$$\therefore I(s) = V_i \times \frac{s}{s^2+5s+4}$$

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Output. $V_0(t) = RI(t)$

$$= 5I(t)$$

Take laplace

$$V_0(s) = 5I(s)$$

$$= 5 \times V_i(s) \times \frac{s}{s^2+5s+4}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{5s}{s^2+5s+4}$$

(a) when $v_1(t) = \delta(t)$; $V_1(s) = 1$

After Partial fraction $V_0(s) = \frac{5s}{(s+1)(s+4)} = \frac{-\frac{5}{3}}{s+1} + \frac{20/3}{s+4}$

impulse response $v_0(t) = -\frac{5}{3}e^{-t}u(t) + \frac{20}{3}e^{-4t}u(t)$

(b) when $v_1(t) = u(t)$

$$V_1(s) = \frac{1}{s}$$

$$V_0(s) = \frac{1}{\cancel{s}} \cdot \frac{5\cancel{s}}{(s+1)(s+4)} = \frac{5}{(s+1)(s+4)}$$

After Partial fraction $V_0(s) = \frac{5/3}{s+1} - \frac{5/3}{s+4}$

$$v_0(t) = \frac{5}{3}e^{-t}u(t) - \frac{5}{3}e^{-4t}u(t) \quad \text{Ans}$$